Test No. 3

(1) Derive a constant-discontinuous discretization in space and energy for the following equation over a single spatial cell, $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$, and the energy domain, $[E_{min}, E_{max}]$:

$$\mu \frac{\partial \psi}{\partial x} + \sigma_t \psi = \int_E^{E_{max}} \frac{1}{4\pi} \sigma_s(E' \rightarrow E)\, \phi(E')\, dE' \; + Q \,.$$

Assume G energy cells, $\{[E_{g+\frac{1}{2}}, E_{g-\frac{1}{2}}]\}_{g=1}^{G}$, and define the basis set so that the unknown in space-energy cell (i, g) is

$$\Psi_{i,g} = \frac{1}{\Delta x_i} \int_{\Delta x_i} \int_{\Delta E_g} \tilde{\psi}\, dx\, dE \,.$$

Assume the trial-space dependence for the inhomogeneous source.

(2a) Derive a linear-discontinuous discretization in space for the following equation over a single spatial cell, $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$:

$$\mu \frac{\partial \psi}{\partial x} + \sigma_t \psi = \frac{\sigma_s}{4\pi} \phi + Q \,.$$

Assume the following trial space on the interior of the cell:

$$\tilde{\psi} = \psi_{i,L} \frac{(x_{i+\frac{1}{2}} - x)}{\Delta x_i} + \psi_{i,R} \frac{(x - x_{i-\frac{1}{2}})}{\Delta x_i} \,.$$

(2b) Repeat problem $(2a)$ using a two-point trapezoidal quadrature for the integrations.

(2c) Use the discretization in (2a) to solve the following problem for $x \in [0, 1]$ with a single cell:

$$\mu \frac{\partial \psi}{\partial x} + \sigma_a \psi = 0\,,$$

with the following boundary conditions:

$$\psi(0, \mu) = f(\mu)\,, \quad \text{for } \mu > 0,$$

$$\psi(0, \mu) = 0\,, \quad \text{for } \mu < 0.$$

(2d) Use the discretization in (2b) to solve the problem given in (2c).

(3) Derive a constant-discontinuous discretization in energy for the following equation:

$$\mu \frac{\partial \psi}{\partial x} + \sigma_t \psi = \int_E^{E_{max}} \frac{1}{4\pi} \sigma_s(E' \to E)\, \phi(E')\, dE' \; + \frac{\partial S \psi}{\partial E} + Q\,,$$

where the stopping power, S is a positive function of energy. Assume the same energy domain and cell structure as in (1), and define the basis set so that the unknown in group g is

$$\Psi_g = \int_{\Delta E_g} \tilde{\psi}\, dE\,.$$

Assume the trial space-dependence for both the stopping power and inhomogeneous source.

* All cross sections are assumed to be constant within a spatial cell.

* Remember to integrate all derivatives by parts so as to avoid having to define a delta-function derivative.