Solutions for Homework 1

Problem 1:

- Equation for ψ^+:

$$\frac{1}{v}\frac{\partial \psi^+}{\partial t} + \frac{\partial \psi^+}{\partial x} + \sigma_a \psi^+ = Q^+ \, . \tag{1}$$

- Solution for $\psi^+(0) = 1$, $Q^+ = 0$:

$$\psi^+(x) = \exp(-\sigma_a x) \, . \tag{2}$$

- Solution for $\psi^+(0) = 0$, $Q^+ = 1$:

$$\psi^+(x) = \frac{1}{\sigma_a} \left[1 - \exp(-\sigma_a x)\right] \, . \tag{3}$$

Problem 2:

- Equation for ψ^-:

$$\frac{1}{v}\frac{\partial \psi^-}{\partial t} - \frac{\partial \psi^-}{\partial x} + \sigma_a \psi^- = Q^- \, . \tag{4}$$

Problem 3:

- Coupled equations for ψ^+ and ψ^- with scattering:

$$\frac{1}{v}\frac{\partial \psi^+}{\partial t} + \frac{\partial \psi^+}{\partial x} + \sigma_t \psi^+ = \frac{\sigma_s}{2}(\psi^+ + \psi^-) + Q^+ \, . \tag{5a}$$

$$\frac{1}{v}\frac{\partial \psi^-}{\partial t} - \frac{\partial \psi^-}{\partial x} + \sigma_t \psi^- = \frac{\sigma_s}{2}(\psi^+ + \psi^-) + Q^- \, . \tag{5b}$$

Problem 4:

- Defining $\phi = \psi^+ + \psi^-$, $J = \psi^+ - \psi^-$, and then adding Eqs. (5a) and (5b), we get

$$\frac{1}{v}\frac{\partial \phi}{\partial t} + \frac{\partial J}{\partial x} + \sigma_a \phi = Q_0 \,, \tag{6}$$

  where $Q_0 = Q^+ + Q^-$.

- Assuming $Q^+ = Q^-$, and subtracting Eq. (5a) from Eq. (5b), we get

$$\frac{1}{v}\frac{\partial J}{\partial t} + \frac{\partial \phi}{\partial x} + \sigma_t J = 0 \,. \tag{7}$$

- Assuming $\frac{\partial J}{\partial t} = 0$, and solving Eq. (7) for J, we get

$$J = -\frac{1}{\sigma_t}\frac{\partial \phi}{\partial x} \,. \tag{8}$$

- Substituting from Eq. (8) into Eq. (6), we get

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial x}\frac{1}{\sigma_t}\frac{\partial \phi}{\partial x} + \sigma_a \phi = Q_0 \,. \tag{9}$$

Problem 5:

- The solution for ψ is

$$\psi(x,\mu) = \frac{1}{4\pi}\left[1 - \exp\left(-\frac{x}{\mu}\right)\right] \quad \text{, for } \mu > 0. \tag{10a}$$

  and

$$\psi(x,\mu) = \frac{1}{4\pi}\left[1 - \exp\left(\frac{(1-x)}{\mu}\right)\right] \quad \text{, for } \mu < 0. \tag{10b}$$

- The rate at which particles about direction μ leave the slab per cm^2 of surface area is

$$\psi(0,\mu)|\mu| \text{ , for } \mu < 0, = \psi(1,\mu)\mu \text{ , for } \mu > 0, = \frac{|\mu|}{4\pi}\left[1 - \exp\left(-\frac{1}{|\mu|}\right)\right] . \tag{11}$$

- The rate at which particles about direction μ are created in the slab is

$$\int_0^1 \frac{Q_0}{4\pi}\, dx = \frac{1}{4\pi} . \tag{12}$$

- The rate at which particles about direction μ are absorbed within the slab is

$$\int_0^1 \psi(x,\mu)\, dx = \frac{1}{4\pi}\left[1 + |\mu|\left(\exp(-\frac{1}{|\mu|}) - 1\right)\right] . \tag{13}$$

Note that leakage plus absorption equals source for each value of μ, and that all particles in direction $\mu = 0$ are absorbed.