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Editor
Dr. Richard F. Deckro
Department of Operational Sciences
Air Force Institute of Technology
AFIT/ENS, Bldg 641
2950 Hobson Way
Wright-Patterson AFB, OH 45433-7765

Publisher
Corrina Ross-Witkowski, Communications Manager
Military Operations Research Society
1703 N. Beauregard Street, Suite 450
Alexandria, VA 22311

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The Military Applications Society (MAS) is a democratically constituted professional society of open membership dedicated to the free and open pursuit of the science, engineering and art of military operations. It is the first society of the Institute for Operations Research and the Management Sciences (INFORMS). MAS advances research in military operations, fosters higher standards of practice of military operations research, promotes the exchange of information among developers and users of military operations research; and encourages the study of military operations research.

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President
Dr. Edward A. Pohl
Department of Industrial Engineering
University of Arkansas
Fayetteville, AR 72701
Phone: 479-575-6042
epohl@engr.uark.edu

Vice President/President Elect
Dr. Patrick J. Driscoll
Department of Systems Engineering
United States Military Academy
Mahan Hall, Room 301
West Point, NY 10996-1779
Phone: 845-938-6587
patrick.driscoll@usma.edu

Secretary/Treasurer
LTC Brian K. Sperling
Department of Systems Engineering
United States Military Academy
Mahan Hall, Room 301
West Point, NY 10996-1779
Phone: 845-938-4399
brian.sperling@usma.edu

Awards Committee Chair and Former President
Dr. Keith N. Womer
Hearin Center for Enterprise Science
University of Mississippi
662-915 5820
kwomer@bus.olemiss.edu

Council Member
Dr. Alan W. Johnson
Air Force Institute of Technology
Dept of Operational Sciences
Phone: 937-255-3636 x 4703
alan.johnson@afit.edu

Dr. Russ R. Vane III
General Dynamics
Phone: 703-450-0137
russ.vane@gd-ais.com

Council Member
LTC Timothy E. Trainor, USA, Ph.D.
Department of Systems Engineering
United States Military Academy
Phone: 845-938-5534
tim.trainor@usma.edu

Dr. Roger C. Burk
Dept. of Systems Engineering
United States Military Academy
Phone: 845-938-4754
roger.burk@usma.edu

Former President
Dr. Richard F. Deckro
Air Force Institute of Technology
richard.deckro@afit.af.mil

Philipp A. Djang
US Army Research Labs
djang@arl.army.mil

Former President
Dr. John P. Ballenger
Raytheon, Inc.
jp_ballenger@raytheon.com

Dr. Bruce W. Fowler
US Army Aviation Missile Command
fowler-bw@redstone.army.mil

Former President
Dr. Tom R. Gulledge
George Mason University
gulledge@gmu.edu

Dr. Steve J. Balut
Institute for Defense Analysis
sbalut@ida.org

Executive Summaries

A HOLISTIC APPROACH TO U.S. ARMY RECRUITING INPUT ALLOCATION

by John R. Brence, Michael J. Kwinn, Jr., and David A. Thomas

Allocating Army resources to meet recruiting mission requirements is a very sensitive and important issue. When the military services meet their recruiting goals, the analytical research focus is on how to keep recruiting constant while reducing inputs such as recruiters and advertising. Conversely, when the military services miss their goals, the analytical research focus is on how to increase recruiting while keeping the inputs constant. Today, we may be faced with the challenge of expanding the military, requiring more recruits and possibly less inputs. This research analyzes the allocation of these inputs to improve recruiting policy and updates the current recruiter allocation model to fit current needs. This paper was selected as Best in Working Group 20: Manpower and Personnel MORSS 2004 and Finalist for 2004 Barchi Prize.

BAYESIAN STOCKPILE RELIABILITY METHODOLOGY FOR COMPLEX SYSTEMS

by Christine M. Anderson-Cook, Todd Graves, Michael Hamada, Nicholas Hengartner, Valen E. Johnson, C. Shane Reese, and Alyson G. Wilson

Logistics require that the U.S. Armed Forces to purchase weapons, ammunition and spare parts in large parts that are stored until the time they are used. In order to effectively understand the health of the stockpiles of these systems, it is important to have statistical models to estimate both current and future reliability at the component and system levels. This paper demonstrates how reliability can be monitored for an aging stockpile with a Bayesian analysis method for a complex system involving multiple components and sub-systems.

REDUCING AIRCRAFT DOWN FOR LACK OF PARTS WITH SPORADIC DEMAND

by Tovey C. Bachman

For sporadically demanded, critical spare parts, traditional methods of relating inventory investment to readiness fail. Setting stock levels for such items challenged operations researchers for over 30 years. In 1999, Tovey Bachman conceived a way of balancing the risks of being out of stock and over-investing in inventory for these parts—the Peak Policy™. Supported by DLA's Aging Aircraft Program, he developed this idea into a simulation-based method for setting stock levels, showing that it could reduce parts shortages and aircraft down as well. DLA began live testing in 2004 and wider implementation is underway. This paper was awarded the 2005 MORS Barchi Prize.

TWO SENSOR MOTION MODELS FOR THE DYNAMIC SENSOR COVERAGE PROBLEM

by Nathan P. Yerrick, Mesut Yavuz and David E. Jeffcoat

The allocation of sensors is a significant problem in surveillance, search, and tracking, with applications ranging from sea to space. The widespread development of unmanned systems provides new opportunities for these platforms to serve as intelligence, surveillance, and reconnaissance (ISR) assets, pulling information from the battlefield into the global information grid. This paper incorporates ideas from automatic control, stochastic processes, and optimization to solve a sensor coverage problem. The solution methods developed might be used to give unmanned air vehicles the capability to make autonomous path planning decisions, or to help a mission planner determine the number of target areas that could be covered by a single ISR asset.

OPTIMAL PLACEMENT OF SUICIDE BOMBER DETECTORS

by Xiaofeng Nie, Rajan Batta, Colin Drury, and Li Lin

There have been an increasing number of suicide bombings on US interests, especially in the Middle East. This article addresses the tactical question of placement of SB detectors in a potential threat area. The objective is the minimization of the number of casualties. The sensitivity analysis allows a decision maker to select the appropriate number of detectors and the appropriate type of detector in the area being protected. The work builds off a recent paper by Kaplan and Kress in the Proceedings of the National Academy of Sciences.

A Holistic Approach to U.S. Army Recruiting Input Allocation

MAJ John R. Brence, Ph.D.

U.S. Army, Computer Emergency Response Team (CERT)
john.brence@us.army.mil

LTC Michael J. Kwinn, Jr., Ph.D.

United States Military Academy
michael.kwinn@usma.edu

Dr. David A. Thomas

Loyaltyworks, Inc.
dave@loyaltyworks.com

APPLICATION AREA:
Manpower and Personnel

OR METHODOLOGIES:
Linear Programming
Nonparametric Stats

ABSTRACT

Development of a useful recruiting model requires an in-depth investigation of previous models and the recruiting processes of today. An objective study of the quantitative and qualitative aspects of recruiting is necessary to meet the future needs of the Army, in light of strong possibilities of recruiting resource reduction and increasing mission requirements. Our research develops a model with an eye towards recruiting process improvement. Our methodology builds on Army recruiting by conducting stakeholder interviews with subject matter experts to construct the foundation of a model that is an efficient starting point for the Recruiter Mission Allocation process, ensures user buy-in, and seeks to fill-in process pitfalls along the way.

Keywords: Recruiting, Resource Allocation, Mathematical Modeling, Data Envelopment Analysis, Regression, USAREC, USAAC

INTRODUCTION

When the military services meet their recruiting goals (early 1990s and today), the analytical research focus is on how to keep recruiting constant while reducing inputs such as recruiters and advertising. Conversely, when the military services miss their goals (late 1990s), the analytical research focus is on how to increase recruiting while keeping the inputs constant. Today, we may be faced with the challenge of expanding the military, requiring more recruits and possibly less inputs.

The key in any environment is the near-optimal allocation of the tight resources available and the reduction in slack resources. In this study, we work with the U.S. Army Recruiting Command (USAREC) and its subordinate Brigades to develop a useful model for allocating recruiters and advertising dollars either optimally or near-optimally.

Historically, the output of the allocation model was taken as a starting point for the final negotiations between commanders within the recruiting community. By the end of lengthy negotiations, the outcomes can bear very little resemblance to the inputs provided by the analysts. At issue is the credibility of the model itself. A measure of our success will be the inclusion of most opinions into the model development, thereby ensuring consensus with the results.

RECRUITER ALLOCATION CONCEPTS

In October 2003, U.S. Army Accessions Command (USAAC) drafted the Recruiting Market Mission Allocation Mathematical Model Statement of Work. This research was tasked to the Operations Research Center of Excellence at the United States Military Academy, West Point, New York. This paper is the result of a year-long research study.

USAAC is revisiting USAREC's recruiter allocation model in order to more effectively and efficiently recruit new soldiers. USAAC/USAREC wants to centrally locate their recruiters in order to maintain proper coverage across the nation and improve recruiter management and productivity.

With a real possibility of an increasing soldier recruiting requirement (mission) and the looming possibility of decreasing the recruiting force and monetary resources, the focus of both the USAAC and the USAREC Commander is to recruit quality soldiers efficiently, while maintaining Army recruiter visibility within all regions of America and mirroring the United States' diverse ethnic demographic. Any model needs to incorporate flexibility in volatile and potential "hotspot" pro-Army recruiting markets in order to focus the efforts of the recruiters and maintain efficiency. In discussion, both commanding generals stated the current "status quo" of recruiting is not a constraining factor; they welcome fresh, new ideas that could position the Army for future success.

Recent studies on recruiting focus on all aspects of recruiting from advertising (Brockett *et al*, 2002) (Dertouzous, 1989) to the recruiting process (Byrnes *et al*, 1988) (Hill *et al*, 2001) (Mehay *et al*, 2000) (Woolever *et al*, 2003) to a complete USAREC overhaul (Marsh *et al*, 1999). The references listed in this document represent only a small sample of research conducted on military recruiting. These studies focused on either qualitative or quantitative methods

to improve recruiting. Qualitative findings on recruiting are quite varied, running the gamut from a total restructure of USAREC to mandatory service of all U.S. citizens (much like Israel). Quantitative research uses models to describe efficiency, to predict resources and describe market propensity. Regression analysis, statistics, Data Envelopment Analysis (DEA), and simulation are common methods used for this research. Our research describes both quantitative and qualitative techniques to prepare for the future of Army recruiting.

U.S. Army Recruiting Command

United States Army Recruiting Command finds its origin in the *Report of the President's Commission on the All Volunteer Armed Force* conducted by Thomas S. Gates, former Secretary of Defense, and his commission, appointed by President Richard Nixon in 1969 (Gates *et al*, 1970). The All Voluntary Army was born, and recruitment of the quality and quantity of soldiers fell directly on USAREC and its predecessors' shoulders. We have not conducted a military draft since the commission reported its findings.

There were many pitfalls in the early days of recruiting, and it was not until 1979, when General Maxwell Thurman grabbed the reigns of USAREC as the Commanding General, that significant measures were taken to improve Army recruiting. Thurman provided USAREC with a new focus, direction, and know-how to create a foundation for successful recruiting. He implemented sophisticated managerial techniques, redefined the USAREC mission, and focused his staff efforts on the "All-Recruited Force". A critical accomplishment of Thurman was convincing Congress to allow paid television advertisements for recruiting in addition to the scarce free public service announcements, formerly the norm (Thurman, 1986).

As of this study, USAREC is meeting its recruiting requirements in all categories; where a category generally equates to male/female and associated threshold for the Armed Forces Vocational Aptitude Test (ASVAB) score. This has not always been the case. At the end of fiscal year 1998, the Department of Defense missed its military recruitment contract mission and there was some concern that a return to the draft was possible. The U.S. Army was short about 17,000 recruits that year (Cavin, 2004). The then-Chairman of the Joint Chiefs of Staff, General Hugh Shelton, testified to the Senate that "he would hate to go back to the draft" (Senate Armed Forces Committee, 1998). General Shelton's comments to the Senate spawned many studies over the years in order to maintain a professional, effective military while avoiding the draft. Recently, in 2005, USAREC failed to meet its recruiting requirements in all categories.

The current USAREC brigade and battalion areas of responsibility are shown in Figure 1.

U.S. Army Accessions Command

The U.S. Army Accessions Command is a fairly new command subordinate to Training and Doctrine Command (TRADOC); established by general order on 15 February 2002. USAAC's mission is to provide integrated command and control of the recruiting and initial military training for the Army's officer, warrant officer, and enlisted forces. The goal of USAAC is to meet the human resource needs of the Army from first handshake to first unit of assignment; this command transforms volunteers into soldiers and leaders for the Army. USAAC is responsible for the management of recruiting enlisted, warrant officers and commissioned officers in both the active and reserve component. USAAC is the parent organization of USAREC.

Most of the recruiting research is coordinated through the USAAC Studies and Analysis Program under the direction of the Center for Accessions Research (CAR). This center was developed to manage recruiting research and house the research library for recruiting studies. The CAR is stationed at USAREC headquarters at Fort Knox, KY.

CLIENT'S INITIAL PROBLEM STATEMENT

According to the original statement of work entitled, "Recruiting Market Mission Al-

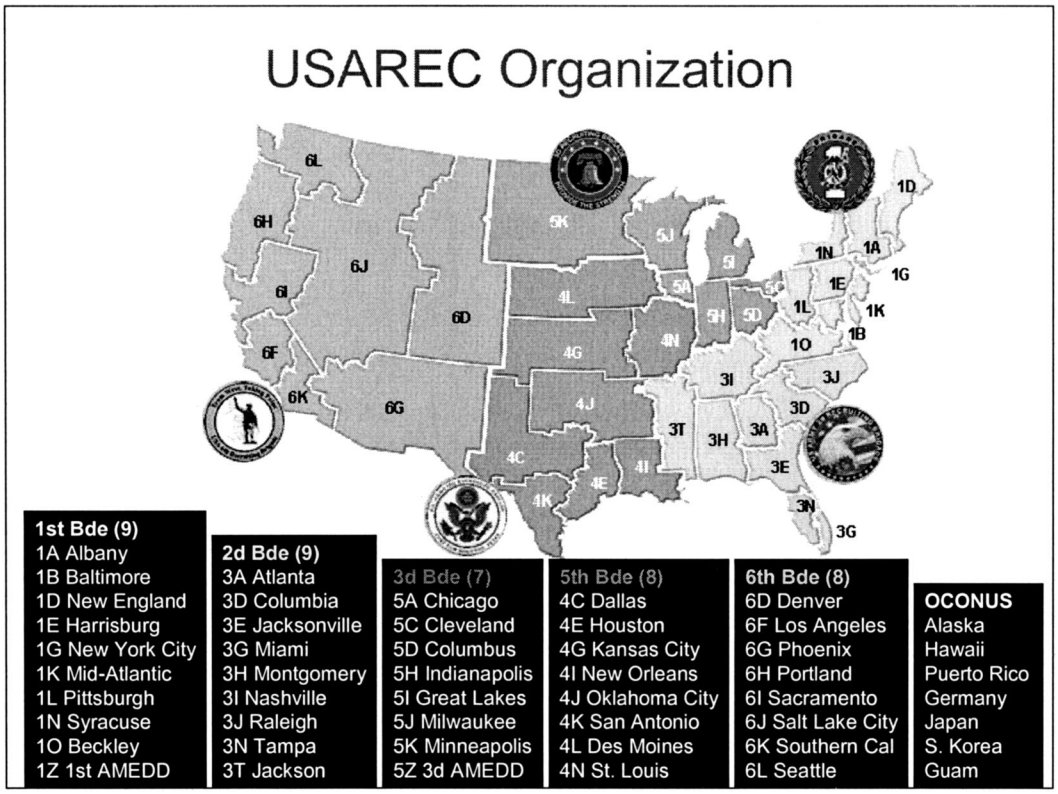

Figure 1. USAREC Brigade Organization.

location Mathematical Model", the client's initial problem statement was two-fold:

- *Develop a model to optimize the placement of recruiters and mission distribution by category.*
- *Use optimization techniques to study, and then develop a mathematical model to optimize territory allocation, placement of recruiter stations, recruiter allocation, and mission distribution with resolution at company level within given agreed upon constraints.*

These tasks solely focused on a mathematical solution to optimize USAREC's recruiting market and recruiting tasks.

RESEARCH METHODOLOGY

Allocating Army recruiters to meet mission requirements is a very sensitive and important issue. Each level of command in USAREC has a key stake in the outcome of this study. This study focused on the CONUS Brigades (Figure 1) to determine the number of recruiters each command will receive. Ideally, each command would like to be heavily resourced with recruiters and lightly burdened with recruitment mission due to the considerable emphasis placed on recruiting mission success. The impact of moving one recruiter or allocating one more recruit to the mission could result in a command failing its mission, which requires a detailed explanation of the reasons for failure directly to the higher headquarters.

The literature review reveals that similar studies were conducted in the past in order to meet changing American demographics, Army Vision, and resource constraints. The ever-changing recruiting environment requires USAREC to periodically reevaluate its allocation model and process to ensure the Army gets the quality and quantity of soldiers needed to defend our nation.

Our approach to this study varies from the previous research. We attempt to systematically capture all the factors in this study using an iterative process known as the Systems Decision Process (SDP). This process was developed at the Department of Systems Engineering at the United States Military Academy, West Point. The process was modified from the Systems Engineering and Management Process (SEMP) developed by McCarthy *et al*, 2003. The revised process was created from a collaborative effort of many individuals over time and the current process is captured in Parnell *et al*, 2006.

Figure 2 is a diagram depicting the flow and iterative nature of the SDP. The SDP consists of four phases, shown as circles, and nine total steps which are named within each phase. The initial and most important phase of our research is Problem Definition.

STAKEHOLDER ANALYSIS

The Stakeholder Analysis is the first step of the Problem Definition. Stakeholder Analysis is important because it assists the researcher in the exploration of the true underlying problem, it helps either to focus or to broaden the scope of the study, and of particular importance, it facilitates user buy-in of the research. By interviewing the key players above and below the decision-maker level, we determine the relevant needs of the system studied. From the needs, wants, and desires of our stakeholders, we can derive the functional requirements and objectives of our study, and in the end, our revised problem statement that focuses on the true crux of the problem.

The use of a systematic approach that includes Stakeholder Analysis is the main differentiation from the previous research. We want to gather information and opinions from the

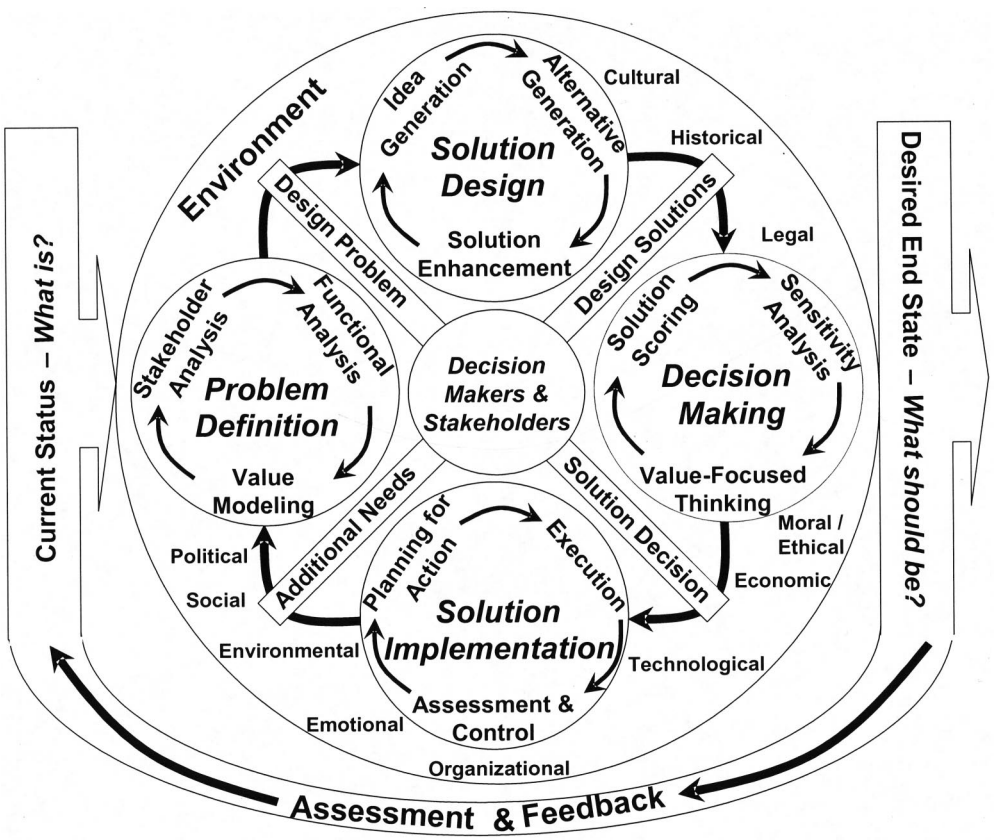

Figure 2. Systems Decision Process (SDP) (Parnell *et al*, 2006).

key players in the recruiting process in order to best address the issue of recruiter allocation and missioning. The current problem statement provided by USAREC is simply to build a better mathematical model to address recruiter allocation, mission allocation, and possibly, recruiting station location (USAREC, 2003). As in any solid analysis, we wanted to first confirm that this statement captured the scope and focus of the issues USAREC wanted to resolve in our study.

Stakeholder Interviews and Interview Questions

We conducted stakeholder interviews with LTG Cavin (Cdr, USAAC), MG Rochelle (Cdr, USAREC), COL Varljen (USAREC CoS), the Recruiting Brigade Commanders, Brigade Headquarters Staff, Brigade Market Chiefs and Brigade Marketing Analysts. In addition, we visited several joint recruiting stations to gain a perspective of allocation impacts at the lowest level and to speak with some of the other service recruiters.

We conducted the interviews prepared with a general question set that was geared to start a discussion. In many of these interviews, we were able to stray away from these questions and discuss more pertinent issues that the stakeholder wished to address. The comments by our prime decision maker are of greatest significance.

Commander, USAREC Comments

One of the toughest questions dealing with recruiting is how to maintain an efficient recruiter presence in a location without losing touch with the community. The Commanders are willing to accept the risk of not having recruiting personnel in every market, if that strategy leads to a better recruiting performance in another area within their command. The question to be answered is which markets have the best and worst propensity for recruiting, especially in light of the current state of the military and the public opinion on U.S. wartime operations.

The USAREC Commander's Intent, from MG Rochelle, was to foster a recruiting environment to attain a significantly increased write-rate i.e. the number of contracts a recruiter writes in a month (Rochelle, 2003). In addition, he commented there is a need to add flexibility to the allocation staffing process and affirmed his willingness to assume risk in areas of the country with low market value i.e. hard to recruit areas. MG Rochelle broadened the scope of our research to provide USAREC with a means, not necessarily a new allocation model, to meet this guidance.

Other Stakeholder Comments

Most of the Brigade Commanders and their staff are concerned with the allocation of recruiters and mission within their command (USAREC Brigade Commanders, 2004). There is much concern about the current allocation process, especially with the mathematical model used to assign recruitment mission and recruiters. The staff process used to determine each Brigade's recruiting mission and the allocation of resources (money, recruiters etc.. . .) to those Brigades is called Recruiter Mission Allocation (RMA). The current RMA process takes up to six months to complete, and the final allocation often bears little resemblance to the mathematical model's solution. The current process starts with the result of the allocation model and then requires a "rebuttal process" by the recruiting brigades. The rationale behind the rebuttal process is that the current model does not do a very good job of predicting recruiting resource needs at the lowest level and fails to provide any insight into future needs or where to assume risk. We understand that no model is perfect; however, we believe that more information may be garnered from a model with predictive ability, not a model based on demographics only. Oftentimes, the demographic data used in this model is not current which is another reason to review the current model and practices.

REVISED PROBLEM STATEMENT

A significant finding from stakeholder analysis is that the revised problem statement

encapsulates two distinct areas. The first area of concern is the modeling aspect of the study that was previously tasked by USAREC. We are to create a model that allocates mission and recruiters to best recruit future soldiers. The second area of interest, gleaned from Stakeholder Analysis, was a need to develop courses of action to update processes and practices of Army recruiting in order to synchronize the mathematical model results with stakeholder needs and desires. This two-pronged approach is essential to provide a holistic solution to the problem and to meet MG Rochelle's intent.

> ***Revised Problem Statement:*** *To develop a flexible and efficient USAREC strategy that improves the enlistment missioning and recruiting process in terms of resource allocation, marketing, and market positioning with an objective to foster a recruiting environment to attain a significantly increased write-rate.*

The revised problem statement is more holistic in nature. This statement is not focused entirely on a mathematical model; it also encapsulates the potential for policy change within USAREC. The combination of a near-optimal mathematical model and the correct recruiting policy adjustments could better perpetuate the recruiting goals determined by the U.S. Army and USAREC.

RECRUITER ALLOCATION MODEL

The current USAREC missioning model is heavily weighted on past recruiting performance, uses limited market analysis for its inputs, and ignores current economic conditions. The model is not predictive in nature and provides no flexibility to the decision-maker in terms of sensitivity analysis. The basis of the model is demographic in nature and assumes that past demographics reflect future demographics. Also, many of the variables are redundant and skew the results to a less than fair share of recruiters to certain brigades, while other brigades are allocated more recruiters with a smaller brigade recruiting requirement. Importantly, the advertising response function has been shown to be Cobb Douglas in many, many studies since the early 1950s. (Allenby *et al*, 2004) (Kao *et al*, 2005) (Marks, 2005) (Thurman, 1988).

Our aspiration is to prevent model misspecification by conducting in-depth research and analysis to find the critical inputs and outputs of such a model. We see the model as a two-step model that first defines the recruiting markets, then allocates recruiters effectively. The outputs of the Market Identification model are the inputs into Resource Allocation model (Figure 3).

From our stakeholder analysis, we have determined that the model needs to include recruiting efficiency and/or propensity to enlist into the Army.

MARKET IDENTIFICATION MODEL

Step one of the two-phased recruiter allocation model is the market identification model. In this step, we created a model to determine which markets, referring to the battalion-level recruiting areas for a brigade, are outperforming any other market. From this model, we can determine the location of pro-Army markets and ensure those battalions are fully resourced in their recruitment effort. This ideology realizes that if a market is not producing Army recruits, theoretically this battalion will not be allocated any resources and essentially shutdown. We will focus our attention on those

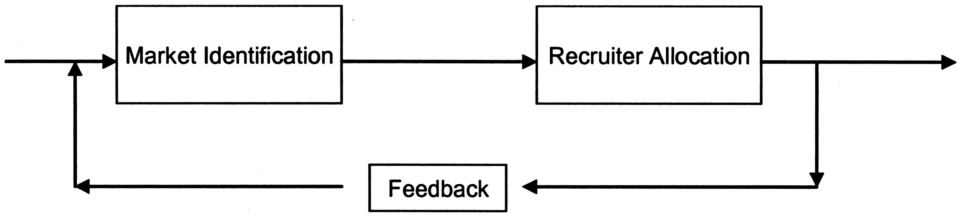

Figure 3. Two-phased Recruiter Allocation Model.

markets that are pro-Army in order to set the conditions for Army recruiting success.

The market identification model uses the same techniques as derived in Brockett *et al*, 2004. Brockett discussed the use of regression coupled with Data Envelopment Analysis (DEA) to determine the best strategy for recruiting advertising mix. His methodology is applicable to this study in that it will enable the user to determine relative recruiting efficiency in the current USAREC markets. By determining recruiting efficiency, we mitigate the effects of managerial inefficiency which allows us to focus our recruiter allocation efforts on the more efficient markets. We chose to model at the battalion level conducting allocations monthly; however, the model may be altered to evaluate longer time frames. Modeling below battalion level provides an increased challenge since current data feeds and census data are relative to battalion recruiting areas.

Regression Model Input to Market Identification Model

From our research, we created the following linear regression model as the initial, simple basis to predict potential recruitment contracts defined as the summation of Graduate Male Category Alpha (GMA) and Prior Service (PS) contracts for a specific area i for month t. We define a potential recruitment contract as the initial signing of a contract by a recruit which may lead to a recruit attending basic military schooling. Past contracts of categories GMA and PS are generally good indicators of a good market for Army recruiting (Rochelle, 2003) (USAREC Brigade Commanders, 2004). Equation (3.1) is the foundation of our market identification model

$$\begin{aligned} Con_{i,t} = \beta_0 + \beta_1 Pop_{i,t} + \beta_2 QMA_{i,t} + \beta_3 Inc_{i,t} \\ + \beta_4 Ump_{i,t} + \beta_5 R_{i,t} + \beta_6 Ad_{i,t} + \beta_7 Con_{i,t-12} \\ + \beta_8 Con_{i,t-3} + \beta_9 Con_{i,t-2} + \beta_{10} Con_{i,t-1} + \beta_{11} t \\ + \varepsilon_{i,t} \quad (3.1) \end{aligned}$$

where the definitions of these variables are shown in Table 1.

The independent variables $Con_{i,t-12}$, $Con_{i,t-3}$, $Con_{i,t-2}$, $Con_{i,t-1}$, and t are included in this equation in order to pick-up any trends, seasonality or the recent recruiting situation in area i. The $Pop_{i,t}$ and $QMA_{i,t}$ variables differ from previous research since they widen the range from 17–21 year olds to the 17–29 year

Table 1. Variable definitions

$Con_{i,t}$	summation of GMA & PS contracts in battalion area $i = 1, \ldots, 41$ for month $t = 13, \ldots, 45$
$Pop_{i,t}$	17–29 year-old male population in battalion area i for month t
$QMA_{i,t}$	17–29 year-old *quality* male population in battalion area i for month t
$Inc_{i,t}$	Median income in battalion area i in month t
$Ump_{i,t}$	unemployment rate in battalion area i for month t
$R_{i,t}$	summation of recruiters Regular Army (RA) and U.S. Army Reserve (USAR) *responsible* for recruiting in battalion area i for month t (Note: partially-missioned recruiters may be expressed as a fraction)
$Ad_{i,t}$	amount of advertisement dollars spent in battalion area i for month t
$Con_{i,t-12}$	summation of GMA & PS contracts in battalion area i for month $t - 12$ (or for the same month of the previous year)
$Con_{i,t-3}$	summation of GMA & PS contracts in battalion area i for month $t - 3$ (or three months earlier in the same year)
$Con_{i,t-2}$	summation of GMA & PS contracts in battalion area i for month $t - 2$ (or two months earlier in the same year)
$Con_{i,t-1}$	summation of GMA & PS contracts in battalion area i for month $t - 1$ (or one month earlier in the same year)
t	observation month (Note: we start with $t = 13$ so it is understood that we need data for the year prior when $t = 1, \ldots, 12$)
$\varepsilon_{i,t}$	error derived from the regression

old male population. The range of ages is extended to 29 years to take into account college students, especially those on the "greater than four year plan" or working and/or struggling their way through school, and to account for the prior service military population that could be recruited into the USAR. The other independent variables were chosen based on past recruiting research (Brockett *et al*, 2002) (Byrnes *et al*, 1988) (Dertouzous, 1989) and the ability of USAREC to obtain this data. Important to note is that we tested the use of interaction terms in the regression equation and found no significant benefit.

Unlike past research, we do not use quotas in this model. These older models used requirements for specific race and/or sex as variables in the recruitment equation. In our case, we do not use such variables; we will utilize DEA to understand and model the impact of "missions" or quotas based on the resultant resourcing of recruiters and dollars to a certain location demographic.

DEA Overview

DEA is a methodology used to separate efficient and inefficient performers. This section provides a brief discussion on those points that are relevant to this study. For a more in-depth discussion on this methodology, see (Cooper *et al*, 2000).

Figure 4 facilitates our discussion of DEA and shows an example of the efficiency frontier created by invoking the DEA methodology. The crux of this methodology is to develop a comparative efficiency policy as determined by the inputs and outputs. The points A, B, C, and D are developed from the data input and output coordinates derived from the envelopment model. The points F and G are more theoretical in nature and their placement in Figure 4 is not discussed in detail.

The solid line segments in this figure create the efficiency frontier. Any point along this frontier is given an efficiency value of 1 or $\theta^* = 1$ in which movement along this frontier entails a trade-off between the input and output values. One can see that a movement from point A to B necessitates an increased level of input to achieve an improved output. This also occurs when moving from B to C.

The extended frontier, shown as the broken line segments terminating with arrows, does not follow the same input-output trade-off as described for the efficiency frontier. In this manner, the movement from G to A results in an output improvement while maintaining the same input level. Likewise, the movement from F to C displays a reduction of the input while maintaining the same output level. Together,

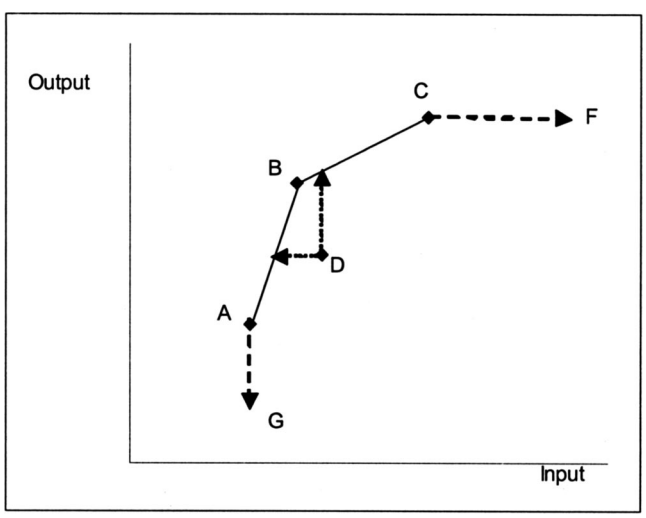

Figure 4. Efficiency Frontier (Brockett *et al*, 2004).

the extended frontier and the efficiency frontier *envelop* all of the other data observations. Because the entire data set (representing the set of production possibilities) lies within the envelope formed from a subset of the data on the efficiency and extended frontiers, we call this method Data Envelopment Analysis.

The point, D, is recognized as an inefficient performer, since its combination of input and output do not place it on the efficiency frontier. In this case we define point D's efficiency rating as $\theta_0^* < 1$ and $(1 - \theta_0^*)x_0$ represents the input reduction needed to eliminate the inefficiencies in D's performance, where x_0 is the current input level. Determining this input reduction leads to increased efficiency.

As an example to highlight this methodology and focus it on our study, suppose D is a recruiting battalion's output for the current month. Since D is not performing efficiently relative to the other battalions on the frontier (A, B or C), then a decision should be made to alter the recruiter and/or advertising dollar inputs to theoretically position D closer to or on the efficiency frontier for the next month.

DEA and Regression

We chose to use a more sophisticated model than Equation (3.1) for two reasons. First, this model does not provide any insight into recruiting efficiency. Our solution to this issue is that recruiting efficiency may be attained through DEA as outlined in the DEA Overview section. Second, the linear relationship of Equation (3.1) does a poor job in emulating the true recruiting markets. For example, a linear relationship between contracts and the number of recruiters is sub-optimal since at some point we would saturate the market with recruiters; in this case more is oftentimes not better. The same argument may be made for advertising dollars.

To more closely emulate the true recruiting markets, we use a logarithmic transform of a Cobb-Douglas production function which from economic theory is said to be *technically efficient* (Cobb *et al*, 1928). Previous research in this area has also utilized this approach (Brockett *et al*, 2002) (Dertouzous, 1989). A Cobb-Douglas function is technically efficient for private sectors, assuming that inefficiency leads to a disbanded company. However, since we are modeling the public sector, where an agency may or may not be successful and still be in business, we need to integrate another means to adjust for efficient performers. We use DEA to make this adjustment.

From this discussion, we can formulate Equation (3.2) to identify the best markets.

$$\begin{aligned} Log(Con_{i,t}) = {} & \beta_0 + \beta_1 Log(Pop_{i,t}) \\ & + \beta_2 Log(QMA_{i,t}) + \beta_3 Log(Inc_{i,t}) \\ & + \beta_4 Log(Ump_{i,t}) + \beta_5 Log(R_{i,t}) \\ & + \beta_6 Log(Ad_{i,t}) + \beta_7 Log(Con_{i,t-12}) \\ & + \beta_8 Log(Con_{i,t-3}) \\ & + \beta_9 Log(Con_{i,t-2}) \\ & + \beta_{10} Log(Con_{i,t-1}) + \beta_{11} Log(t) \\ & + D_{EFF_{i,t}}[\gamma_0 + \gamma_1 Log(Pop_{i,t}) \\ & + \gamma_2 Log(QMA_{i,t}) + \gamma_3 Log(Inc_{i,t}) \\ & + \gamma_4 Log(Ump_{i,t}) + \gamma_5 Log(R_{i,t}) \\ & + \gamma_6 Log(Ad_{i,t}) + \gamma_7 Log(Con_{i,t-12}) \\ & + \gamma_8 Log(Con_{i,t-3}) \\ & + \gamma_9 Log(Con_{i,t-2}) \\ & + \gamma_{10} Log(Con_{i,t-1}) + \gamma_{11} Log(t)] \\ & + \varepsilon_{i,t} \end{aligned} \tag{3.2}$$

where γ_j are coefficients for each input to account for the efficient and inefficient performers. These coefficients and corresponding variables are "activated" by the variable $D_{EFF_{i,t}}$ which returns a value of 1 for an efficient performer and 0 for inefficient performers from our DEA analysis (Brockett *et al*, 2002). All of the other variables were discussed in detail in Table 1.

In our case, elasticity refers to the impact of the number of contracts based on a marginal increase or decrease of a resource variable. From the market identification model, the elasticity for an efficient performer for a given input is identified by $\beta_j + \gamma_j$. Similarly, the elas-

ticity for an inefficient performer for a given input is simply β_j. For example, when operating efficiently, the elasticity for the population variable ($Pop_{i,t}$) will be $\beta_1 + \gamma_1$, whereas when operating inefficiently, the elasticity is β_1 (Brockett *et al*, 2002).

Equation (3.2) was formulated through a two-stage approach as proposed by (Brockett *et al*, 2002). The first step was to run DEA on the raw data to distinguish between efficient and inefficient performers. The second step was to run a regression with the logistic transform of the independent and dependent variables using the entire dataset merged with the observation values for efficient performers discovered from DEA. The conglomeration of this technique is shown in Equation (3.2) where the portion of the equation related to the β_j's is the regression function of the entire dataset and the portion starting with $D_{EFF_{i,t}}$ is derived from DEA and only incorporates data from the efficient performers. This model is the basis for the objective function in Equation (3.3).

It is important to note that any observation for variable input into this model cannot be zero. The log transform is undefined for zero values. Therefore, the size of the recruiting area investigated should not be smaller than the size defined by recruiting station boundaries; this assumes that at least one quality recruit is contracted by each recruiting station each month. We chose to model at the battalion area level to alleviate any such problems; this approach aligned with USAREC's wishes.

RECRUITER ALLOCATION MODEL

The second phase of this model is the allocation of recruiters to the best markets. In order to be more precise, we combined like variable inputs from Equation (3.2) for the objective function, i.e. $\beta_j + \gamma_j$ for all variables. In addition, we transform these components back to a Cobb-Douglas production function using the exponential transform. Our model formulation for the prediction of a specific battalion monthly contracts ($C\hat{o}n_{i,t}$):

$$Max \text{ GMA + PS Contracts } (C\hat{o}n_{i,t})$$

$$= \sum_{i=1}^{I} e^{\beta_0+\gamma_0} Pop_{i,t}^{\beta_1+\gamma_1} QMA_{i,t}^{\beta_2+\gamma_2} Inc_{i,t}^{\beta_3+\gamma_3} Ump_{i,t}^{\beta_4+\gamma_4} R_{i,t}^{\beta_5+\gamma_5} Ad_{i,t}^{\beta_6+\gamma_6} Con_{i,t-12}^{\beta_7+\gamma_7} Con_{i,t-3}^{\beta_8+\gamma_8} Con_{i,t-2}^{\beta_9+\gamma_9} Con_{i,t-1}^{\beta_{10}+\gamma_{10}} t^{\beta_{11}+\gamma_{11}} \quad (3.4)$$

Subject to:

$$\sum_{i=1}^{I} R_{i,t} \leq C_1$$

$$C_1 = \text{\# recruiters available for the month} \quad (3.5)$$

$$R_{i,t} \leq C_2;\ i = 1 \ldots I$$

$$C_2 = \text{max recruiters per area} \quad (3.6)$$

$$\sum_{i=1}^{I} Ad_{i,t} \leq C_3$$

$$C_3 = \text{total advertising budget for the month} \quad (3.7)$$

$$Ad_{i,t} \leq C_4;\ i = 1 \ldots I$$

$$C_4 = \text{max advertising monies per area} \quad (3.8)$$

$$\sum_{i=1}^{I} C\hat{o}n_{i,t} \geq C_5$$

$$C_5 = \text{USAREC monthly recruiting mission} \quad (3.9)$$

where all variables are non-negative. Variables are previously defined in Table 1.

Equation (3.5) constrains the number of recruiters assigned at or below the total number of recruiters available for duty for the current month. Equation (3.6) constrains the number of recruiters assigned at or below some maximal value per recruitment area for the current month. Equation (3.7) constrains the amount of advertisement dollars at or below the total advertising budget for the current month. Equation (3.8) constrains the amount of advertisement dollars spent on a particular recruiting

area at or below some maximal amount for the current month. Equation (3.9) ensures that our potential contracts meet or exceed the USAREC recruitment mission for the current month for a specific recruiter and ad dollar allocation. In addition, we adhere to the requirement that all decision variables used in this mathematical program are non-negative.

This formulation will position recruiters (and advertisement dollars) in recruiting areas; it does not delineate between a RA and USAR recruiter. It places a recruiter in a market based on the total number of recruiters available (RA + USAR). If the location of USAR recruiters is a concern, we suggest that USAR recruiters are assigned against allocation totals in areas closest to USAR centers. The number of USAR recruiters could be varied based on the size of the USAR center.

DATA ANALYSIS AND MODELING RESULTS

To exercise our theoretical approach, we attained data for the variables listed in Table 1 from USAREC's databases and the 2000 Census. These data are the foundation of our model since the information contained within the data provides us with insight of which recruiting battalions are working efficiently during a specific month. Our model uses data from the period FY01 to 3d Qtr, FY04. The FY01 data segment is used only for past contract information ($t = 1\ldots12$). The bulk of the data modeled resides in FY02 to 3d Qtr, FY04 ($t = 13\ldots45$) and from this data; we develop an advertising and recruiter allocation plan for July 04 ($t = 46$).

Data Properties and Descriptions

The dependent variable, $Con_{i,t}$ is the number of GMA+PS contracts in battalion area $i = 1, \ldots, 41$ for month $t = 13, \ldots, 45$. Graduate Male Category Alpha (GMA) are male recruits that graduate high school with a high recruitment quality rating. Prior Service soldiers (PS) are counted because these recruits are vital to the USAR mission. The GMAs and PSs are added together to create a single value for a specific battalion in a specific month.

$Pop_{i,t}$ and $QMA_{i,t}$ are differing 17–19 year old male population variables in that $Pop_{i,t}$ includes all 17–29 year old males and $QMA_{i,t}$ only maintains those 17–29 year old males that meet the quality standards as imposed by USAREC. We only had yearly data available for these variables, so each month in a specific year had the same value. There were; however, different values for each of the 41 battalions. Albeit these variables appear to be collinear, we retained them in the model at the request of several primary stakeholders.

$Inc_{i,t}$ is the median household income for a specific battalion area in a certain month. This specific data was not available to us, so we had to construct this data from the 2000 U.S. Census by zip code median household income tables. This data was calculated forward to our modeled time period using historic monthly inflation data found at *www.inflationdata.com*. The median household income for each battalion was then derived from the previously calculated monthly zip code data for a specific month.

$Ump_{i,t}$ is the unemployment rate as provided by USAREC. The $Ump_{i,t}$ was available for each battalion for each month.

$R_{i,t}$ is the sum of RA and USAR recruiters recruiting within a battalion area in a specific month. This information was also available from USAREC for each battalion for each month.

$Ad_{i,t}$ is the amount of advertising dollars allocated and used by a specific battalion in a month. The data we received from USAREC was a quarterly summary for each fiscal year. In order to get the data into a monthly format, we simply divided the quarterly monies into equal monthly amounts.

$Con_{i,t-12}$, $Con_{i,t-3}$, $Con_{i,t-2}$ and $Con_{i,t-1}$ are calculated in the same manner as the dependent variable $Con_{i,t}$ except for a different month; twelve months prior, three months prior, two months prior and one month prior, respectively.

t is the observation month. Our data observations range from $t = 1\ldots45$, using $t = 13\ldots45$ as the bulk of the modeled data, and we allocate advertising money and recruiters for $t = 46$.

DEA Analysis

We conducted DEA analysis on 33 months of data (t = 13. . .45) for 41 battalions or 1,353 observations. To determine the efficient observations, we used the "max out" optimization mode (to maximize the output given the current inputs) coupled with the varying scale mode (outputs fall off as input levels rise). The varying scale mode is also referred to as the BCC (Banker, Charnes and Cooper) model (Banker *et al*, 1984). In our study we use these DEA model specifications because Army recruits are in limited supply and we cannot assume that even if we receive more recruiters and advertising dollars that the number of recruits will increase in direct proportion.

From our analysis, we calculated 274 efficient performers out of 1,353 observations (see Figure 5). The efficient performers are derived evaluating each battalion within a certain time period as a single market entity.

These 274 efficient performers' data values are re-included in the dataset as efficient values in order to prepare for the regression Equation (3.2) i.e. these performers score a 1 for the $D_{EFF_{i,t}}$ and all others score a 0 (inefficient performers).

DEA and Regression Analysis

Armed with the knowledge of which battalion is an efficient performer in a specific month, we are now able to run our market identification model highlighted in Equation (3.2). Table 2 highlights our results of DEA and Regression analysis. This table depicts the variable significance for the entire dataset (column 1) and for those determined by DEA as efficient (column 2). The efficiency coefficient column depicts the coefficients (previously discussed in section 0 as $\beta_j + \gamma_j$) denoting the elasticity for an efficient performer.

Of note, it is interesting to see that AD$ is not significant at the 95% level in either case; yet for β_6 it is significant at the 90% confidence level. Normally we would eliminate this variable from inclusion in our market identification model; however, we maintain AD$ in our model based on our stakeholders' desires. The intercept of this equation is also significant at the 90% confidence level. In addition, in both cases, the Inc, Con_12, and Con_1 are significant at the 95% confidence level, hinting that some past contract performances influence ef-

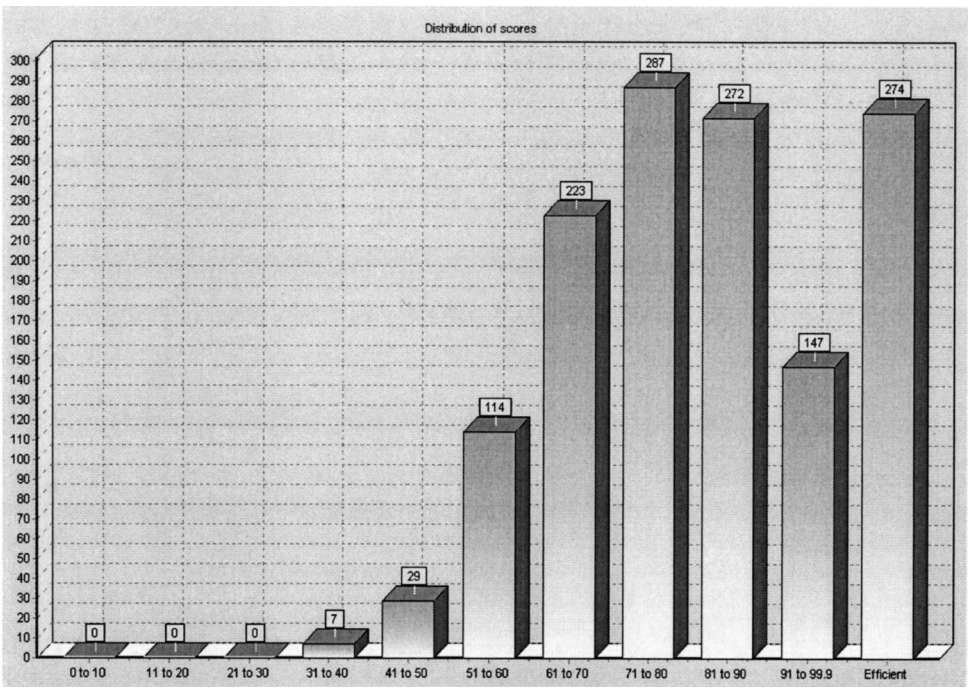

Figure 5. DEA Efficiency Summary Chart (output from Banxia Software, 2003).

Table 2. Significant variables from DEA and regression analysis

Variables	All Data+		DEA Data		Efficiency Coefficients	
Pop	β_1	0.2922*	γ_1	−0.0684	$\beta_1 + \gamma_1$	0.2238
QMA	β_2	−0.3501*	γ_2	0.1247	$\beta_2 + \gamma_2$	−0.2253
Inc	β_3	0.1331*	γ_3	−0.1813*	$\beta_3 + \gamma_3$	−0.0482
Ump	β_4	0.1611*	γ_4	0.0085	$\beta_4 + \gamma_4$	0.1696
Recruiter	β_5	0.2483*	γ_5	0.0322	$\beta_5 + \gamma_5$	0.2805
AD$	β_6	0.0036	γ_6	0.0018	$\beta_6 + \gamma_6$	0.0054
Con_12	β_7	0.1563*	γ_7	0.2256*	$\beta_7 + \gamma_7$	0.3819
Con_3	β_8	0.0782*	γ_8	0.0275	$\beta_8 + \gamma_8$	0.1057
Con_2	β_9	0.2685*	γ_9	0.0575	$\beta_9 + \gamma_9$	0.3260
Con_1	β_{10}	0.2706*	γ_{10}	−0.1240*	$\beta_{10} + \gamma_{10}$	0.1466
Time	β_{11}	−0.1024*	γ_{11}	0.0078	$\beta_{11} + \gamma_{11}$	−0.1102
Intercept	$\beta_0 + \gamma_0$				−0.6118	

* Statistically Significant at 95% level.
+ Includes inefficient performers.

ficient and inefficient performers as does the median income of the battalion area.

Recruiter Allocation Model Results

From the DEA and Regression results, we use the efficiency coefficients for the objective function for our allocation model formulated previously as Equation (3.4) from the results of Table 2. The objective function equation is shown below as Equation (4.1). As an example resource allocation, we apply this objective and constraints outlined as Equations (4.2) to (4.6) for the 41 battalions for the month of July 04 ($i = 1...41$, $t = 46$).

$$Max \text{ GMA} + \text{PS Contracts } (\hat{Con}_{i,t})$$

$$= \sum_{i=1}^{41} e^{-0.6118} Pop_{i,46}^{0.2238} QMA_{i,46}^{-0.2253} Inc_{i,46}^{-0.0482} Ump_{i,46}^{0.1696} R_{i,46}^{0.2805} Ad_{i,46}^{0.0054} Con_{i,34}^{0.3819} Con_{i,43}^{0.1057} Con_{i,44}^{0.3260} Con_{i,45}^{0.1466} 46^{-0.1102} \quad (4.1)$$

Subject to:

$$\sum_{i=1}^{41} R_{i,46} \leq 6{,}350 \text{ recruiters} \quad (4.2)$$

$$R_{i,46} \leq 300 \text{ recruiters}; i = 1...41 \quad (4.3)$$

$$\sum_{i=1}^{41} Ad_{i,46} \leq 900{,}000 \text{ dollars} \quad (4.4)$$

$$Ad_{i,46} \leq 900{,}000 \text{ dollars}; i = 1...41 \quad (4.5)$$

$$\sum_{i=1}^{41} \hat{Con}_{i,46} \geq 7{,}667 \text{ recruiting contracts} \quad (4.6)$$

where all variables are non-negative.

The values used on the right-hand-side of the constraint equations are reasonable estimates for available resources. For Equation (4.2), the 6,350 recruiters depicted is the summation of RA and USAR recruiters available for duty. This total was adjusted from 6,296 the month prior based on the projected increase in recruiters based on mission requirements. The 300 recruiters shown in Equation (4.3) assumes that each battalion can only manage at most 300 recruiters in their area based on interpolation of past data provided by USAREC and a capacity increase to allow efficient battalions to receive more recruiters. The monthly total advertising cap of $900,000 in Equation (4.4) is derived from past USAREC advertising trends the year prior. The battalion area advertising cap of $100,000 in Equation (4.5) is calculated from past data provided by USAREC and a capacity increase to allow efficient battalions to receive more advertising monies. Equation (4.6) en-

sures that the contract estimation from the model meets the minimum monthly contract needs. The monthly recruiting contract mission of 7,667 is based on a yearly mission of 80,000 recruits, divided monthly, with a 15% buffer to protect against failed contracts.

We used a genetic algorithm to solve this model and calculate several allocations of recruiters and advertising dollars in order to maximize the monthly contract estimate. Our best feasible allocation is shown in Table 3.

This allocation meets all constraints and estimates a surplus of 5,101 contracts based on the goal of 7,667 for this month. At the Brigade level, the allocation is summarized in Table 4.

The allocations in this example are consistent with the current trends of Army recruiting. America's Heartland (3rd BDE area) normally produces the least recruits while the West Coast (6th BDE area) produces the most. The southern states (2nd and 5th BDE areas) normally provide a good recruiting market, while the northeastern states (1st BDE area) are close behind. Our allocations work to support successful recruiting, not to provide equal resourcing as desired by the USAREC Commanding General.

Our modeling effort emulated these current trends because DEA unveils efficient performers by extracting the important input variable values based on the data modeled. However, this quantitative result is only valid in a "perfect world". Our quantitative analysis assumes that recruiter and populace behavior will follow the same data trends that were modeled. It would be an obvious error to place total confidence in such a result; therefore, to ensure a complete analysis of the problem at hand, we divert our attention to some qualitative aspects of recruiting such as policies, procedures and leadership issues.

RECRUITING PROCESS ANALYSIS

Our decision to study the process of recruiting from the bottom-up is heavily based on our Stakeholder Analysis. Initially this phase of our research was not in the scope of the study; however, it is very clear that there are many qualitative issues that plague USAREC and Army recruiting. There was a significant concern about the bureaucracy of choosing recruiting station location and how leases kept recruiters in a potentially stagnant market. Other comments indicated that the leadership was inexperienced with recruiting, since most individuals only spend a three-year tour on task. Lastly, many stakeholders explained that there was a lack of effort or an "overwhelming" effect on new, inexperienced recruiters. Many of these comments led us to believe that a decent mathematical model would only solve a portion of the problem and more analysis of the process was necessary.

The most significant research effort we accomplished was to gather a panel of experts to discuss the future of Army recruiting. Our panel was comprised of many former personnel who served in USAREC and/or were tasked to conduct similar studies. Many of these individuals have gone on to be leaders of industry in related fields such as human resources and marketing. We received expert advice and feedback on what research was done in the past and cutting-edge methods used today by industry to recruit and market. In this analysis, we evaluated potential areas for recruiting process improvement and came up with several suggestions for USAREC.

The purpose of this workshop was to develop attainable courses of action that focus on improving USAREC's current recruiting management process. We focused our effort on MG Rochelle's intent; to develop courses of action to update processes and practices of Army recruiting in order to achieve a 2.0 write-rate by FY06, provide recruiting flexibility and management of recruiting risk in weak markets.

The workshop participants worked in various divisions within USAREC during the 1980–2000 timeframe. Many of these individuals worked as division heads as well as in the recruiting trenches from Brigade and below. Included were two individuals who provided insight specifically on USAR recruiting. Many of these people have moved on to marketing and human resource companies that conduct similar analysis.

Table 3. Example battalion allocation and projected contracts for July 04

BN ID	BN Name	July 04 Allocations		Projected Contracts
		AD$	Recruiters	
1A	Albany	$4,182.48	50	126
1B	Baltimore	$12,421.53	268	599
1D	New England	$2,692.72	140	299
1E	Harrisburg	$4,827.64	101	166
1G	New York City	$8,792.30	163	358
1K	Mid-Atlantic	$7,359.36	124	258
1L	Pittsburgh	$6,495.90	117	194
1N	Syracuse	$8,449.74	130	226
1O	Beckley	$5,953.93	102	195
3A	Atlanta	$4,756.15	135	246
3D	Columbia	$4,439.23	169	330
3E	Jacksonville	$9,984.62	138	286
3G	Miami	$11,923.62	146	301
3H	Montgomery	$14,852.04	171	320
3I	Nashville	$11,553.56	123	242
3J	Raleigh	$16,962.75	213	454
3N	Tampa	$11,414.12	135	285
3T	Jackson	$7,495.88	92	147
4C	Dallas	$24,284.66	275	575
4E	Houston	$15,705.36	211	433
4G	Kansas City	$16,921.05	156	303
4I	New Orleans	$9,207.62	132	248
4J	Oklahoma City	$12,376.46	177	315
4K	San Antonio	$19,877.66	154	269
4L	Des Moines	$30,014.86	113	205
4N	St. Louis	$31,481.30	192	415
5A	Chicago	$26,380.04	158	303
5C	Cleveland	$47,550.23	134	203
5D	Columbus	$39,498.07	142	258
5H	Indianapolis	$29,019.91	158	352
5I	Great Lakes	$36,462.37	150	341
5J	Milwaukee	$42,385.43	134	277
5K	Minneapolis	$40,756.73	119	237
6D	Denver	$34,910.67	127	244
6F	Los Angeles	$43,130.32	233	478
6G	Phoenix	$38,833.54	251	554
6H	Portland	$36,796.55	171	343
6I	Sacramento	$36,315.06	181	403
6J	Salt Lake City	$42,170.44	147	288
6K	Southern Cal	$51,171.19	173	358
6L	Seattle	$40,192.93	145	334
TOTAL		$900,000.00	6,350	12,768

RECRUITING PROCESS RECOMMENDATIONS

From the initial stakeholder analysis and analysis of the data from the Recruiter Management Workshop, we uncovered several promising alternatives. Of note were significant modifications to recruiting management and policy that pertain to the Army's accessions strategy.

Table 4. Example brigade allocation and projected contracts for July 04

BDE NAME	July 04 Allocations		Projected Contracts
	AD$	Recruiters	
1st	$61,175.60	1,195	2,421
2nd	$93,381.96	1,322	2,611
5th	$159,868.97	1,410	2,763
3rd	$262,052.78	995	1,971
6th	$323,520.70	1,428	3,002

Growing and Finding Recruits

The primary comment from the group was to "fish where the fish are," a drastic change to Army recruiting which assumes risk in weak recruiting markets. Those failing markets could easily be covered by part-time recruiters, kiosks or by advertising (primarily through the internet). We can keep in touch with America through the internet and part-time help. Through our discussion, we continually expressed that "All markets are not the same. Some regions just won't make it." The idea is to locate recruiters in strong recruiting markets and maintain coverage in the weak markets by other means; an example of economy of force.

Recruiter Management

We determined to be successful in recruiting, the Army needs to be more active in making a positive presence in a community. Using community influencers i.e. teachers, coaches, police officers as part time recruiting liaisons will allow the Army to push its message to a much younger population. These people would not have a recruiting mission but are compensated based on accessions they refer. These part-timers are our early contact force (that does not require the brick-and-mortar station to operate). This alternative does not shrug off recruiting college students, which presumably will always provide quality recruits; it just assists the Army in developing a positive foothold in a growing community.

Some pearls of wisdom gained from the experienced team focused more on management of recruiting personnel at all levels. Most of the focus was on the recruiter in the trenches and how we select, train, motivate, manage, and reward them. In our interviews with many detailed recruiters, those recruiters that are taken from other Army specialties to fill-in shortages of professional recruiters, we found that they were very adamant about "getting out of recruiting ASAP!" A course of action that may increase sales and provide a quicker way out of recruiting for detailed recruiters is to provide an overall goal for the three-year recruiting tour (tour mission), where once that recruiter meets that cumulative goal, they return to the Table of Organization and Equipment (TOE) Army. Backfilling this soldier is the only tricky part of this suggestion; however, it may also provide flexibility and timeliness to position the replacement recruiter in a station that is in more dire need. This assumes a Brigade is constrained by the current number of recruiters.

Another incentive related course of action is to treat recruiting Special Duty Assignment Pay (SDAP) similarly to flight pay. Everyone gets SDAP when entering recruiting, but continuation depends upon meeting accession gates at periodic reviews. The accession gates could be calculated based on monthly write-rate or quarterly accessions. These reviews could be done yearly.

Recruiter selection and skills management is vital to maintaining an effective selling force. We need to identify recruiters with the mentality and ability to sell ~ and keep them selling. A good salesman could be a recruiting specialist; maybe not a "hard-stripe" soldier. Because many of these recruiters are not management material, we should not consider the station manager position as a step toward promotion. The recruiting business is the closest the Army gets to a profit-focused organization; it should be structured a bit differently. Likewise, we should consider using a permanent professional manager as station commander. This manager may be a civilian or a permanent 79R (military professional recruiter). Structuring management in this fashion would maintain local area knowledge and continuity.

Many of the problems in recruiting are tied to the detailed recruiting company commanders, typically Captains, because they lack moti-

vation to work in recruiting, are in a "last-chance to fix their Army career" status or are not capable of handling the complex nature of the job. As a form of career progression for 79Rs, we could commission them as warrant officer recruiting company commanders. This would keep experienced and effective station managers in recruiting trenches, but at a higher level of responsibility (and pay). Training to prepare the soldier for the new duties should be minimal based on their previous experience.

In addition, a strategic reserve recruiting force, coined as the Elite Recruiting Force (ERF), was discussed to provide flexibility to the commander either at the USAREC or Brigade level. The ERF would consist of the most experienced salesmen and a few managers. The ERF would be established at either the USAREC or Brigade level. This elite force would not have a recruiting mission; they would provide recruiting flexibility to the commander. If the ERF was managed properly, they could be used to 1) re-train stations (on-site) that are under-performing, 2) conduct spot-checks on the recruiting practices of stations, 3) provide flexibility to assist a station with an extremely "hot market", 4) plan and conduct special events to promote the Army and 5) be deployed to meet a re-emerging market, then work to maintain a more permanent presence. This idea is similar to the agricultural concept of "resting" a farm before seeding it again.

Lastly, we discussed outsourcing cold-calling lead generation to a telemarketing firm. Let the recruiters recruit and manage the Delayed Entry Program (DEP). To get a soldier to assess, there are many tasks a recruiter needs to accomplish; one of the most time consuming and important tasks is to become a physical trainer for the new recruit. Cold-calling keeps the recruiter tied to his desk in which he loses visibility with the market outside of the recruiting station.

USAREC Staff Tasks and Policies

Probably the most resounding commentary from our research was for USAREC to do a better job of knowledge management. Many of these studies and/or programs discussed in our workshop have been previously implemented or researched. There were two courses of action that may assist USAREC in knowledge management: 1) Establish a professional operations research cell at USAREC so that the past lessons are retained and 2) maintain a professional staff in USAREC set-up like a for profit business with traditional departments such as marketing and sales. Both the operations research cell and the professional staff should be permanent party thereby maintaining domain knowledge. If necessary, these positions may be civilianized.

Another key idea is to set-up a recruiting career field for officers and/or warrant officers. Creating an institutional support career field in recruiting could improve the promotion chances for soldiers in the recruiting field and hopefully dismiss the perception that serving in USAREC is bad for an Army career. For officers, this should work the same way as other career fields work in the Army. The officer will start out as a functional area designated recruiting officer, and then at the Majors' promotion board, the officer will select their top choices for a career field. The intention is to place officers and warrant officers in this field who actually enjoy recruiting and/or the ability to live almost anywhere in/out of the U.S.

To maintain currency with a changing market, USAREC needs to buy all of the data needed for market segmentation and RMA processes. USAREC should make a long-term contract with a first tier data provider. This comment is critical to maintaining a grasp on the ever-changing recruiting environment. By gathering the most up-to-date data, USAREC can maintain flexibility to shift recruiters to emerging or re-emerging markets; utilizing the ERF if necessary. Money saved from assuming risk around the country should be spent on better market and data analysis.

Professional Research and Support Group

One of the most important results of this research was gaining nine more subject matter experts as research sources. These gentlemen are very experienced in the field of recruiting

and recruiting planning and were able to give excellent insight on the improvement of the Army's recruiting system. In addition, they have provided us with a network of individuals who could answer any further questions related to recruiting.

In 2004, The SAS Group announced a "commitment to sponsor a program designed to bring together retired senior military personnel with leaders in the private sector and academia to study and facilitate the adoption of best business practices by the military." (SOURCE SAS Institute Inc., 2004). We suggest creating a more permanent Panel of Recruiting Experts to assist in the future direction of Army recruiting. These gentlemen have expressed an interest in continuing service to USAREC even though retired. They have provided an extensive amount of information that has application in recruiting today. From the discussions in our workshop, they expressed that they were forced to tackle the same problems in recruiting during their tenure and could provide insight on their successes and failures. This would be an excellent technique to help improve knowledge management at USAREC.

CONCLUSIONS AND FUTURE WORK

Our focus in developing a recruiter allocation model was to keenly study the required inputs to develop an efficient, feasible model that closely describes what is required for USAREC to meet or surpass its recruiting mission. We are very sensitive to the needs of the people involved in this process and feel that the model needs "user buy-in" to be effective. The current and previous models were never validated with any confidence, even though USAREC still made mission. Most of the success of USAREC lies in its leadership and hard work from all individuals involved. We would like to lessen the burden of the RMA process and set-up each command level for success by creating an effective model and recommending several process improvements.

The difficulty in the derivation of this model is deciding how to succinctly build it so all parties understand how and why it works, while taking into account the accuracy of the model. The model should be useful enough that only slight modifications are made to the recommended recruiter resourcing. The benefits of such a model are that it would lessen the duration of the RMA process and decrease the workload of the leadership. Ideally, as the model continues to evolve and the leadership becomes more confident in the recruiter allocation model, the RMA process will focus only on the model result with insignificant feedback from the recruiting brigades.

The integration of quantitative and qualitative aspects of this research could lead to an efficient, forward-looking recruiting community. We understand that a quantitative model is not perfect representation of the real world or future; nevertheless, insights from a good model lead to better policy decisions from those commanders and staff officers saddled with the hard resourcing decisions. Such mathematical models assist the decision maker by providing a basis for an educated decision without him having to thumb-through terabytes worth of data. Policy decisions generally make-or-break any market strategy in terms of output. Policy decisions are often made by and are responded to by individuals based on qualitative aspects e.g. motivation and desires. If motivation or desire is lacking, in either the recruiter staff and/or recruit, then the Army would be less likely to meet its recruiting requirements. In terms of efficiency, it is more beneficial to provide a quantitative model to provide resourcing guidelines for a motivated recruiter selling to a motivated recruit. This is the ideal situation; one that we aspire to see in the future.

Future directions for this research would require a test application of the modeled allocation results and validation of the recommended process improvements. This analysis is important to ensure the real-world significance of the variables, even though past research has shown statistically these variables are important. Implementation of this model coupled with the aforementioned process improvements, we believe that Army recruiting will be successful, especially now, when our country needs a strong and responsive military.

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Bayesian Stockpile Reliability Methodology for Complex Systems

Christine M. Anderson-Cook
Los Alamos National Laboratory
c-and-cook@lanl.gov

Todd Graves
Los Alamos National Laboratory
tgraves@lanl.gov

Michael Hamada
Los Alamos National Laboratory
hamada@lanl.gov

Nicholas Hengartner
Los Alamos National Laboratory
nickh@lanl.gov

Valen E. Johnson
U.T. M.D. Anderson Cancer Center, Unit 447
vejohnson@mdanderson.org

C. Shane Reese
Brigham Young University
reese@hannibal.byu.edu

Alyson G. Wilson
Los Alamos National Laboratory
agw@lanl.gov

APPLICATION AREAS: Air and Missile Defense; Logistics, Reliability, and Maintainability
OR
METHODOLOGIES: Reliability; Bayesian Networks

ABSTRACT

Bayesian reliability modeling of complex systems, such as a missile system, can allow considerable flexibility to incorporate component and subsystem level data, expert knowledge and full system tests. In this paper we present a unified method for developing a model that is able to consistently estimate reliability parameters for all elements of the model, from system level through to subsystem and component levels. The model can be adapted to model the various components of the system at different levels of granularity, depending on where data are available. In addition, this paper presents some direction about how Bayesian priors can be selected to incorporate expert knowledge of the system in a variety of ways. An example of a complex system based on a missile is used to illustrate the methods.

INTRODUCTION: A BAYESIAN APPROACH TO STOCKPILE RELIABILITY

Logistics require the U. S. Armed Forces to purchase weapons, ammunition, and spare parts in large lots that are stored until the time they are used or retired. The management and maintenance of such stockpiles, which are collections of similar complex systems, face many issues related to the aging of the stored units. Commonly, the units degrade to a point that they may fail to function as intended. The central problem of *stockpile reliability* is to estimate what fraction of units within the collection of similar systems will work. In other words, we are interested in determining the probability that a *randomly* selected unit from the stockpile will function.

Current strategies for assessing the reliability of a stockpile rely on testing the success rate of a sample of units from the stockpile. For expensive and complex systems, the number of sampled units for these destructive full-system tests, which are considered the most direct measurements of the success of the system, may necessarily be small. But many other types of tests, from component tests to visual inspection to functional tests of specific subsystems, are possible and can provide indirect information that can complement the full-system testing. Typically, the relative proportion of data available from these alternate sources may dramatically outnumber the full-system tests, and hence their inclusion into analysis methods can enhance prediction and precision of estimation substantially. The challenge is to develop a methodology that allows for the integration of various sources of data and information to assess the performance and reliability of the units in the stockpile.

Ideally, we would like to have enough full-system tests to be able to accurately assess the reliability of a stockpile. But due to cost and other considerations, this may not always be possible. Fortunately, there are often other sources of information available about system reliability. For example, complex systems can often be described in terms of interrelated subsystems and components, and data from component and subsystem tests may be available. Here, not only is the information about the reliability of each piece of the full-system important, but also the specific nature of how these pieces are connected to each other to produce functionality in the system. (The distinction between a subsystem and a component may not always be obvious. For this paper, components are the smallest subunits we want to test and subsystems are portions of the entire system which represent collections of related components.) Other sources of information also include engineering knowledge related to the components, the subsystems and/or the full system, visual inspections and other non-destructive testing, such as electronic diagnostics and battery checks.

The Statistical Sciences Group at LANL continues to develop a suite of methods and tools, collectively known as Information Integration Technology (IIT), that allow the combination of information from various sources to predict performance and reliability for complex systems with limited or no data from full-system tests. In this paper, we describe one application of IIT to the assessment of performance, reliability and shelf life of units within a stockpile that combines full-system tests with component tests and engineering judgment. The key aspects of this approach that will be explained in some detail in this paper are (1) a unified method for consistently allows for

prediction at the system and component levels, and (2) how expert judgement and other indirect sources of knowledge about the system can be incorporated into a single model through informative Bayesian priors to maximize the information summarized in the overall model.

By modeling the reliability of each of the systems, using data from the entire stockpile, we can answer not only, what is the probability that a particular unit will perform as intended, but we can also give point and interval estimates for the expected proportion of working units in our stockpile. Since the units are all from the same population, the center and spread of the distribution of the stockpile reliability can be estimated as a function of the individual probabilities of success.

APPLICATION: MISSILE OVERVIEW

Background and System Model

The methodology presented has been applied to a particular missile, but for clarity of presentation, we have simplified some details of the system. We precisely describe the modeling and statistical procedures followed, and preserve all of the features of the system that provided statistical challenges for the analysis.

Some of the key characteristics of the system presented include the following: First, the system can be logically decomposed into subsystems, which are natural groupings of components focused on a particular task of the mission. Some of the subsystems combine directly to form the system, while other subsystems may be nested within subsystems. The form of the system representation is largely driven by where sources of data are available and how the engineers best feel the system functions can be summarized. Second, data are available at some, but not all, of the components and subsystem. Third, there are several variants of the missile. Treating each variant as possibly having different reliability can increase the precision of the estimation process relative to grouping together nonhomogeneous variants (for example, systems which do not share all the same components or were produced by different manufacturers), and help provide insights into which variants are more likely to fail.

For the example presented in this paper, we consider a system with three major subsystems with a total of 13 components. Figure 1 shows an event tree diagram of how subsystems and components combine to form the system. For more information about event trees, see Roberts (1987).

The components and subsystems are connected by AND gates, implying that all components and subsystems need to function for the full system to function. The system is labeled as C_1, comprised of a subsystem C_2 (made up of two components, C_5 and C_6, and a subsystem C_7 with 8 components, C_{10} to C_{17}), a component C_3, and a subsystem C_4 (made up of 2 components C_8 and C_9). As mentioned previously, the labeling of individual parts of the system as components or subsystems is somewhat subjective and is dependent on what information is available. The key factor in determining what level of detail should be included in the summary of the system and subsequent analysis is the sources of data available. For example, "component" C_3 is actually comprised of a collection of "subcomponents", which might have been alternately represented as a "subsystem" with "components", if data had been available on the subcomponents.

The system shown in Figure 1 has only 17 components with 4 levels of nesting of compo-

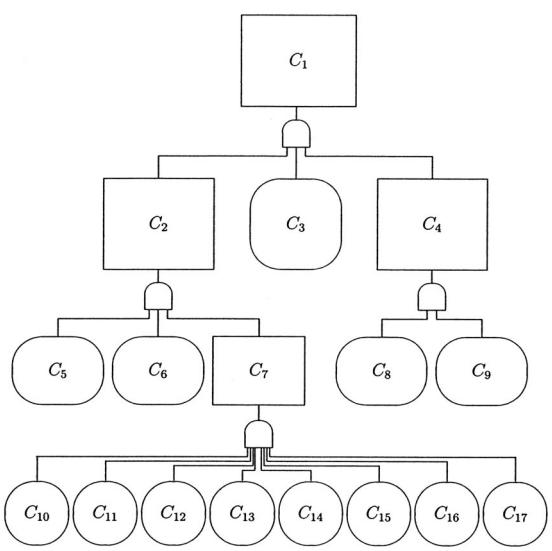

Figure 1. Event Tree Diagram for Missile Reliability

nents within subsystems and system, and as such does not represent an overly complex system. This methodology has been employed by the Statistical Sciences group at LANL for considerably larger problems, but for simplicity we have used this relatively simple example. The methodology outlined in the remainder of the paper is scaleable to much larger systems with more components and levels of nesting, with the understanding that the computational demands for estimation and prediction increase dramatically as the system becomes larger.

Data Description

In estimating the reliability of a complex system, it is common to have test data and prior engineering judgment available at the system, the subsystem, and the component levels. Methodology for combining these various sources of information in a consistent fashion has proven problematic (Bier (1994)), and the goal of this paper is to describe an IIT approach that resolves this difficulty. For simplicity, we restrict discussion to systems in which components or subsystems are classified as either functional or not (pass/fail).

Four sources of data are considered here. The first consists of full-system data consisting of pass/fail results from 1249 flight tests.

The second is data collected from component or subsystem tests, also assumed here to take on only pass/fail values. Several collections of observations are available, ranging from 70 to 100 observations for some, but not all, of the components and subsystems. Some of the data have covariate information, including storage location and the age of the system at test, which will not be used in this analysis. Anderson-Cook et al. (2005) consider the case where the system level covariate, age, is used to model degradation over time.

Third, engineering judgment regarding the probability that a specific component or subsystem fails may be available. It represents substantial subjective knowledge about the working of the system and its intended design that helps to bound the reliability.

A fourth, less precise source of information is engineering judgment stating that a group of components in a given system or in related systems have similar failure probabilities. For example, an expert may assert that the reliability of the missile battery is "similar" to the reliability of a battery in a related missile system, or that reliabilities of types of motors are similar. Alternately, an expert may judge that all of the components of a given subsystem are equally likely to cause the failure of that subsystem. This says that the reliability of the components are thought to be similar, not that the failure mechanisms of the components are the same.

BAYESIAN RELIABILITY MODELING

Background

The new methodological advances presented in this paper primarily involve how Bayesian models can be adapted to give consistent system down and component up representation of success/fail reliability models given different granularity of the modeling, and some insights into how meaningful and informative Bayesian priors can be formed to incorporate multiple sources of indirect supplementary information. To provide context, it is useful to begin with a review of some related research in Bayesian system reliability.

As with all statistical models used in system reliability, the first issue is the specification of the system structure, which formalizes how components and subsystems are related to each other and to the entire system. Many of the common ways of making this specification are graphical and include reliability block diagrams, event trees, fault trees, and Bayesian networks. For more details on any of these approaches see Rausand and Høyland (2004). Each of these representations implies a particular structure for the way that data are incorporated to make inferences about the system as a whole—this structure provides a means of obtaining the *likelihood*, which represents the information in the data.

Often the likelihood contains unknown parameters. In classical or frequentist statistics, the standard approach is to find the value of the parameters that maximizes the likelihood of the

observed data. These estimated parameter values can then be used in conjunction with the fitted model to obtain predictions and to make other inference. Bayesian methodology allows one to augment the data with expert engineering knowledge via the specification of an a priori distribution for the unknown parameters, called the *prior distribution*. The data are then used to update the prior distribution on the parameters to obtain a *posterior distribution* that captures both the information from the prior and from the observed data. As with classical inference, the posterior distributions allow prediction and inference. As the amount of observed data increases, the results from Bayesian and classical inference tend to give increasingly similar estimation and prediction results, since information added through the prior distribution is downweighted relative to the data's contribution.

Raiffa and Schlaifer (1961) consider the fundamental problem of modeling binomial distributions with beta priors. Martz, Waller and Fickas (1988) and Martz and Waller (1990) studied similar system reliability problems with binomial data, and also faced the problem of integrating expert opinion at different levels. They assumed that they were supplied with beta distributions for the reliabilities of individual components and of the subsystems made up of those components. Beta distributions for the components are then used to derive "induced" distributions for subsystems (for example, a series subsystem is modeled as a product of random variables with the components' beta distributions). This "induced" distribution is then combined with the subsystem's beta distribution, and the combination is itself approximated using a beta distribution, with parameters determined by the method of moments. This procedure is repeated until the full system level is reached. Graves and Hamada (2004) reanalyze these data sets using the same Bayesian methods as in the present paper, which do not require approximations. A shortcoming of the Martz et al. (1988) method is that the full system data do not inform estimates of single component reliability. Because of this and since full system tests suggested lower reliability than the component and subsystem tests would indicate, Graves and Hamada (2004) obtained substantially smaller reliability estimates for some components.

Many reliability models do not consider prior expert opinion and data at multiple system levels. Springer and Thompson (1966, 1969) and Tang, Tang and Moskowitz (1994, 1997) provide exact or approximated system reliability distributions obtained by propagating the component posteriors through the system structure. Thompson and Chang (1975), Chang and Thompson (1976), Lampkin and Winterbottom (1983), and Winterbottom (1984) use approximations for exponential lifetimes rather than binomial data. Others propose methods for evaluating or bounding moments of the system reliability posterior distribution (Cole (1975), Mastran (1976), Dostal and Iannuzzelli (1977), Mastran and Singpurwalla (1978), Barlow (1985), Natvig and Eide (1987), and Soman and Misra (1993)). These moments can also be used in the beta approximations employed by Martz et al. (1988) and Martz and Waller (1990). Soman and Misra (1993) proposed a distributional approximation based on a maximum entropy principle.

The Binary System and Component Model

The model developed here assumes that the status of each component, subsystem and full system is in just one of two states: pass or fail. This status of the tested unit is the simplest outcome of a reliability test.

The restriction to binary status implies that independent testing results in a binomial likelihood. The representation of the full system as connected subsystems and components in series makes the status of the system and subsystem functionally dependent on the status of the components. As a result, the parameters of the binomial likelihood of subsystems and the full system are functions of the parameters of the components. By coding a success as a one and a failure as a zero, the outcome is modeled by a Bernoulli random variable Y with distribution governed by a single parameter, the probability p it takes value one (namely, a success). The probability of a failure, the only other possible outcome, is therefore, $1 - p$. The goal

of the modeling is to use the available data to estimate the parameters p_i for each component, subsystem, or full system, C_i, with their associated uncertainties. Primary interest likely lies in obtaining an estimate of the probability that the overall system C_1 succeeds, namely p_1.

As discussed previously, combining data and prior information at different levels within a reliability diagram has often proven problematic, both from the perspectives of computational tractability and model consistency. Our solution to this dilemma is to simply re-express nonterminal (subsystem) node probabilities in terms of terminal (component) node probabilities using deterministic relations derived from the system reliability diagram. For example, from Figure 1, it is evident that the probability, p_7, that subsystem C_7 functions is equal to the product of the probabilities, p_{10} through p_{17}, that each of the components C_{10} through C_{17} all function. Thus,

$$p_7 = \prod_{i=10}^{17} p_i.$$

Similarly, the probability that subsystem C_4 functions is $p_4 = p_8 \times p_9$.

Note that variable substitutions based on the reliability diagram do not uniquely identify a joint distribution on the terminal node probabilities, in this case p_{10} through p_{17}. However, this approach does use the marginal distribution of the system reliability to provide a sensible, although not necessarily unique, solution for the joint distribution of all of the component and subsystem reliabilities. See Johnson, Graves, Hamada and Reese (2003) for additional details.

Prior Specification

In many applications, engineering judgment can play an important role in assessing system reliability, particularly for large complex systems where data collected on individual components is sparse. Judgment and expertise are always required to develop the system representation and determine which data are relevant to the analysis.

In this analysis, we explore the statistical modeling appropriate for two additional types of expert judgment:

(1) Precise Information: The expert provides precise information about the reliability of a single component by specifying a probability distribution for the failure probability of the component.
(2) Component Groupings: The expert identifies a component as belonging to a group of components with similar reliability. This assumption does not require that the components are physically similar, only that their reliabilities are similar. For example, all the high reliability components might be judged to be similar. We model this type of information via a hierarchical model for the failure probability. Technically, this corresponds to the expert providing us with a family of probability distributions for the failure probability.

Precise Information. Engineering judgment may be available from several experts, and the quality of information obtained from each expert may vary. In our model, we therefore assume that the prior density obtained from expert m concerning a specific value of p_i takes the form of a beta density, and we let the set of combinations of (i, m) for which engineering judgment is available be denoted by S_1. A beta density is a flexible class of probability distributions that assigns mass between 0 and 1, thus making it suitable to capture judgment about probabilities. See Gupta and Nadarajah (2004) for more details on the beta distribution and how the choice of parameters influences the mean and variance of the distribution.

More specifically, we assume that the prior information is modeled as:

$$\frac{\Gamma(N_m + 2)}{\Gamma(N_m \pi_{i,m} + 1)\Gamma[N_m(1 - \pi_{i,m}) + 1]} p_i^{N_m \pi_{i,m}} (1 - p_i)^{N_m(1 - \pi_{i,m})} \equiv \mathrm{B}(p_i; N_m \pi_{i,m} + 1, N_m(1 - \pi_{i,m}) + 1). \quad (1)$$

Note that this is not the standard parameterization of the beta distribution, but is selected for the desirable feature that it has the

mean of the distribution as one of the parameters, $\pi_{i,m}$ and N_m corresponds to the degree of belief that $\pi_{i,m}$ is the actual reliability. To equate the more standard B(a, b) distribution with mean $a/(a + b)$, set $a = N_m\pi_{i,m} + 1$ and $b = N_m(1 - \pi_{i,m}) + 1$. See Gupta and Nadarajah (2004) for details.

In equation (1), $\pi_{i,m}$ represents expert m's point estimate of p_i, and N_m represents the estimated precision of expert m. The parameter N_m is unknown, and for concreteness, we assume that each expert precision parameter N_m is drawn from a gamma density with known parameters α_m and β_m, parameterized here as

$$G(N_m; \alpha_m, \beta_m) = \frac{\beta_m^{\alpha_m}}{\Gamma(\alpha_m)} N_m^{\alpha_m - 1} \exp(-\beta_m N_m).$$

In summary, engineering judgment that a particular parameter value is approximately $\pi_{i,m}$ can be captured by a beta distribution. By specifying a prior distribution for N_m, we can allow additional flexibility to be added to adjust the variability of the estimate and use Bayes theorem to produce a posterior distribution of N_m given the data, thereby allowing us to assess empirically the expertise of the expert. We expect that if an expert's opinion is inconsistent with the data, then N_m will be small, in effect downweighting that experts contribution to the inference.

Note that engineering judgment has the form of a binomial likelihood with a maximum at $\pi_{i,m}$. This convention eliminates the possibility that the joint density specified on all model parameters is improper, and also implicitly handles the aggregation problem identified by Bier (1994) by simply treating engineering judgment as a form of "data."

Simple examples of the differences between fixed and random N are illustrated in Figure 2. In each case, the data are x successes in $n = 50$ binomial trials with unknown success probability p. The prior distribution for p is $B(N\pi, N(1 - \pi))$ for $\pi = 0.5$, and we compare the case where

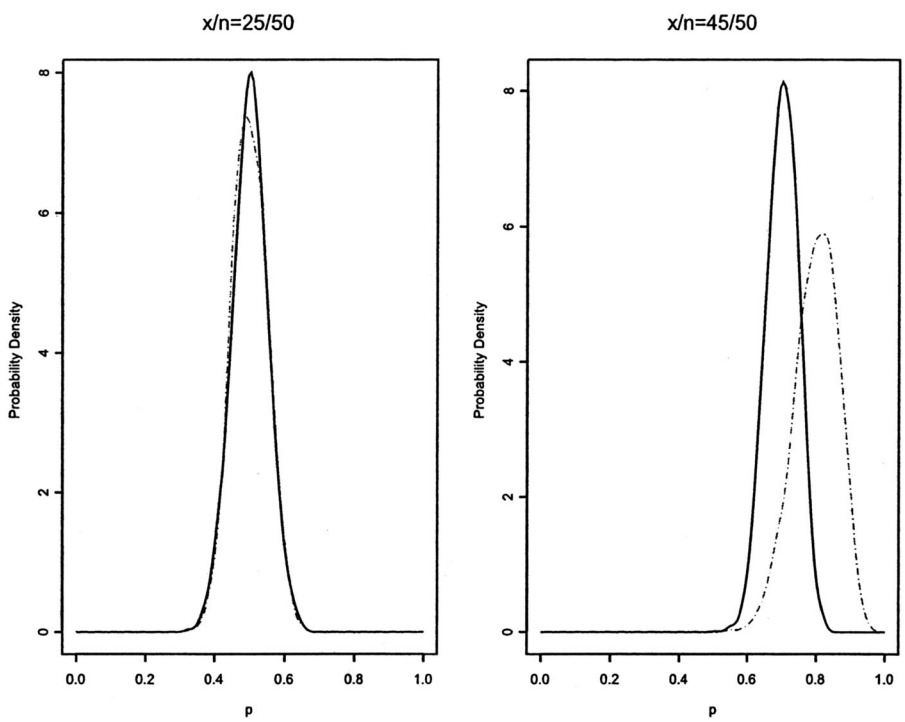

Figure 2. Comparison of fixed and random prior sample sizes. Shown are posterior densities of p with a $B(N\pi, N(1 - \pi))$ prior with $\pi = 0.5$ based on $x = 25$ (left) or $x = 45$ (right) successes in $n = 50$ trials, for the cases of fixed $N = 50$ (solid curves) or random N with a prior distribution with mean 50 and SD 22 (dotted-dashed curves).

N is fixed at 50 with the case where N has a gamma prior distribution with parameters $\alpha = 5$ and $\beta = 1/10$ (so that $E(N) = 50$). In each plot, the solid curve is a posterior density estimate for p in the fixed N case, while the dotted-dashed curve is a density estimate for p in the random N case. The left plot is for the case of $x = 25$ successes, which is exactly consistent with the prior mean of $\pi = 0.5$. In this case, the data are not very informative about N so that its posterior is close to its prior; the expert could be very reliable (large N) or unreliable and fortunate to have guessed the right value (small N). The posterior mean of N is 52, slightly larger than its prior mean of 50, and its posterior standard deviation (SD) is about the same as its prior SD of 22.4. The posterior mean of p is 0.5, and its posterior SD is essentially 0.05 in either the fixed or random N case. Therefore no harm is done by allowing N to be random in the case of data that agree with the prior. However, if the data and the prior disagree, as in the right plot of Figure 2 with $x = 45$, the random N analysis gives different results. For fixed $N = 50$, the inference is like 70 successes in 100 trials so $E(p) = 0.7$ and $SD(p) = 0.046$. When N is random, its posterior mean of N is 18 with SD 11, so that the prior for p is deemphasized and we get $E(p) = 0.8$ and $SD(p) = 0.063$; the mean is closer to the high reliability implied by the data, while the standard deviation is larger because the prior is contributing less data. The fixed N analysis is misleading because the accuracy of the prior has been overestimated, as is shown by its disagreement with the data, and the random N analysis behaves accordingly, treating the prior as useful information but not as accurate as initially thought.

Component Groupings. When prior information regarding component success probabilities are not always known, it is often possible for experts to assign components into groups of "similar reliability". The reliabilities of components within a given group are not necessarily the same, but they are exchangeable. Hence, equation (1) is augmented in the model by assuming that $\pi_{i,m}$ is replaced by $\rho_{m,g}$, where $\rho_{m,g}$ represents the common, but unknown, success probability assigned by expert m to components in the broader group g. The form of the prior on model parameters from such information takes the form

$$\prod_{(i,m)\in S_2} \mathrm{B}(p_i; K_m\rho_{m,g} + 1, K_m(1 - \rho_{m,g}) + 1).$$

Here, S_2 denotes the combinations of (i, m) for which such grouping information is available.

As in equation (1), the parameter K_m is assumed to be drawn *a priori* from a gamma distribution having parameters ζ_m and η_m. The prior success parameter $\rho_{m,g}$ for a particular group is assumed to be from a beta distribution with known parameters $\delta_{g,m}$ and $\epsilon_{g,m}$, respectively. Here $\rho_{m,g}$ can be interpreted as a common mean estimate of the individual p_i's for elements within that group.

This approach allows for knowledge from different components to be leveraged across components deemed to be similar, to increase predictive power and exploit common features in the system.

Hierarchical Model to Address Data Granularity. In this analysis, a hierarchical prior is used on the components/terminal nodes. This prior is used not to capture expert judgment, but to render estimates of the overall system reliability insensitive to the level of detail included in the system event diagram.

As an illustration of this point, consider a simple system comprised of three components connected in series (similar to a system, C_1, comprised only of C_2, C_3 and C_4, with no other components) and suppose that a single binomial observation with four successes and one failure is observed at the system level. Without a hierarchical specification on the component probabilities and under the model assumptions stated above with uniform priors, the likelihood of the system reliability would be proportional to

$$(p_2p_3p_4)^4(1 - p_2p_3p_4)$$

where $p_1 = p_2p_3p_4$ given the assumed independence of failures of each of the components and the series layout that requires all three components to work for the system to operate. This portion of the likelihood is obtained using the fact that since four of the trials resulted in successes, implying that components C_2, C_3 and C_4 all worked, which occurs with probability

$p_2p_3p_4$. The trial which resulted in a failure had probability, $1 - P(Success) = 1 - p_2p_3p_4$.

If we assume a uniform distribution for the prior of each of p_2 to p_4, the posterior mean of p_1 in this model is 0.507. When the system is not decomposed into subsystems and a uniform prior is assumed on p_1, the posterior mean on p_1 (with a uniform prior) is 0.714. Furthermore, under such naive model specifications, the bias attributable to adding components to the event tree becomes more severe as the number of components in the system increases.

Suppose instead that the reliabilities of components C_2, C_3, and C_4 are not assumed to have independent uniform distributions. Instead, each component's success probability is drawn from a beta distribution (as given in equation (2) with parameters $J\gamma$ and $J(1 - \gamma)$, where the parameter J is assumed drawn from a gamma density function with parameters τ and ϕ. The parameter γ is assumed to be drawn from a beta distribution with parameters ψ and ω.

$$\frac{\Gamma(J)}{\Gamma(J\gamma)\Gamma[J(1-\gamma)]}p_i^{J\gamma}(1-p_i)^{J(1-\gamma)} \equiv \mathrm{B}(p_i; J\gamma, J(1-\gamma)). \tag{2}$$

Using this hierarchical prior specification on p_2 to p_4 with $\psi = \omega = 0.5$ results in a posterior mean of 0.718 for p_1, while the same specification with $\psi = \omega = 1.0$ results in a posterior mean of 0.687. Both estimates are largely insensitive to the number of components specified for the system. This is an important feature of this model, as the availability of data for different components and subsystems in the system should not play an important factor in influencing overall reliability estimate of the system.

Joint Distribution. To obtain a posterior distribution for the Bayesian analysis, the likelihood which contains the information captured in the data must be combined with the expert judgement summarized in the prior distributions. The posterior is proportional to the product of the likelihood times the prior distributions. For more details, see Martz and Waller (1982). Combining the information from the likelihood and priors leads to a joint posterior distribution on the model parameters proportional to

$$\begin{aligned}
&[\boldsymbol{p}, \mathbf{N}, \boldsymbol{\rho}, \mathbf{K}, \gamma, J \\
&\qquad |\mathbf{x}, \mathbf{n}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \boldsymbol{\delta}, \boldsymbol{\epsilon}, \boldsymbol{\tau}, \boldsymbol{\phi}, \boldsymbol{\psi}, \boldsymbol{\omega}] \propto \\
&\prod_{i\in S_0} p_i^{x_i}(1-p_i)^{n_i} \\
&\times \prod_{(i,m)\in S_1} \mathrm{B}(p_i; N_m\pi_{i,m} + 1, N_m(1-\pi_{i,m}) + 1) \\
&\times \prod_{m:\exists(i,m)\in S_1} \mathrm{G}(N_m; \alpha_m, \beta_m) \\
&\times \prod_{(i,m)\in S_2} \mathrm{B}(p_i; K_m\rho_{m,g} + 1, K_m(1-\rho_{m,g}) + 1) \\
&\times \prod_{m:\exists(i,m)\in S_2} \mathrm{B}(\rho_{m,g}; \delta_m, \epsilon_m) \times \prod_{m:\exists(i,m)\in S_2} \mathrm{G}(K_m; \zeta_m, \eta_m) \\
&\times \prod_{i\in S_0} \mathrm{B}(p_i; J\gamma, J(1-\gamma)) \\
&\times \mathrm{B}(\gamma; \psi, \omega)\mathrm{G}(J; \tau, \phi).
\end{aligned} \tag{3}$$

In equation (3), we have an expression for the joint posterior distribution of the parameters of primary interest ($\boldsymbol{p}$, $\mathbf{N}$, $\boldsymbol{\rho}$, $\mathbf{K}$, γ, $\mathbf{J}$), which are shown to the left of the vertical line on the left hand side of the expression. The vertical line indicates that we are conditioning on the other parameters of the model ($\mathbf{x}$, $\mathbf{n}$, $\boldsymbol{\pi}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\eta}$, $\boldsymbol{\delta}$, $\boldsymbol{\epsilon}$, $\boldsymbol{\tau}$, $\boldsymbol{\phi}$, $\boldsymbol{\psi}$, $\boldsymbol{\omega}$) to obtain an expression that is proportional ($\propto$) to the joint posterior distribution. S_0 denotes the set of terminal nodes/components.

In this expression, values of nonterminal node probabilities (such as those for elements C_1, C_2, C_4 and C_7) are assumed to be expressed in terms of the appropriate functions of terminal node probabilities, as defined from the system event diagram. The first line of equation (3) corresponds to the contribution from observed data for each available component to the likelihood function. The second and third lines correspond to the contribution of expert opinion for individual components through the component priors, while lines 4 and 5 summarize the information from experts on groups of components.

An examination of the contributions to the joint posterior distribution arising from the prior information reveal obvious similarities, but there are also important distinctions be-

tween these parameterizations. For example, the value of N_m represents the precision of the expert's opinion, while K_m describes the similarity of item reliabilities within a grouping.

Implementation: Markov Chain Monte Carlo

The joint posterior distribution in equation (3) does not have a familiar distributional form, so it is not immediately obvious how to do inference and estimation. Recent advances in Bayesian computing allow one to obtain a random sample from the joint posterior distribution. The general approach used is called Markov chain Monte Carlo (MCMC) and it permits the simulation of any distribution on a finite-dimensional state space specified by any unnormalized density. Once one has a random sample obtained from the MCMC algorithm, then inference can be made on any of the quality characteristics of interest. There are several variations of MCMC which are appropriate for this type of problem. Gibbs sampling (Casella and George (1992)) is one method to draw random samples from the joint posterior distribution. Another method is the Metropolis-Hastings algorithm (Chib and Greenberg (1995)).

One issue with MCMC sampling is whether the draws are approximately a random sample from the posterior distribution; this is referred to as the convergence of the MCMC sampler. To mitigate the impact of initial values chosen for the parameters, a burn-in is typically performed in which the MCMC sampler is run a number of times and the draws thus obtained are discarded. To reduce dependence between draws, the draws can be thinned by retaining every kth draw. See Raftery and Lewis (1996) for more discussion of diagnostics for convergence.

The joint distribution on model parameters previously specified does not lead to analytical evaluation of the system or component reliabilities. However, a componentwise Metropolis-Hastings algorithm can be implemented in a relatively straightforward way. In our version of such a scheme, we use a random-walk Metropolis-Hastings algorithm with Gaussian proposal densities specified on the logistic scale for the terminal node probabilities, as well as for ϱ_0 and $\rho_{m,g}$. Precision parameters are similarly updated using a random-walk Metropolis-Hastings scheme with Gaussian increments specified on the logarithmic scale. The resulting Metropolis-Hastings algorithms are implemented using YADAS (Graves (2001) and (2003)).

Example: Posterior and Predictive Distribution

Using engineering judgment, reliability classes are formed as follows. System elements C_2–C_4 are assigned to Group 1, elements C_5–C_9 to Group 2, and C_{10}–C_{17} to Group 3. Note that there is great flexibility about the creation of groups, as some of them include both components and subsystems; also, the groupings need not be mutually exclusive. Beta distributions with common, fixed group means (π_1–π_3) and a single, common precision parameter ($N_{1,2,3}$) are assumed for each of Groups 1–3. A common precision parameter is incorporated for each group, since a single expert provided all this information. The individual components in this system are C_3, C_5, C_6 and C_8–C_{17}. A hierarchical prior with unknown mean and precision parameter (γ and J, respectively) is assumed for components in Group 4. Also, gamma priors with parameters (5,1) are assumed for both precision parameters ($N_{1,2,3}$ and J), and a diffuse prior ($\psi = \omega = 0.5$) is assumed for γ.

Applying the model previously discussed, we obtained the posterior distributions on the component reliabilities for each of the components and the expert precision parameters. The system reliability posterior distributions with the system data included and system data excluded are plotted in Figure 3. It is clear that when the flight test data are included a much more precise estimate of system reliability can be obtained, as noted by a much narrower posterior distribution. However, it should also be noted that the posterior distribution obtained without the benefit of the flight data, and only based on the indirect component and subsystem data, is able to appropriately estimate a sensible range of system reliability values.

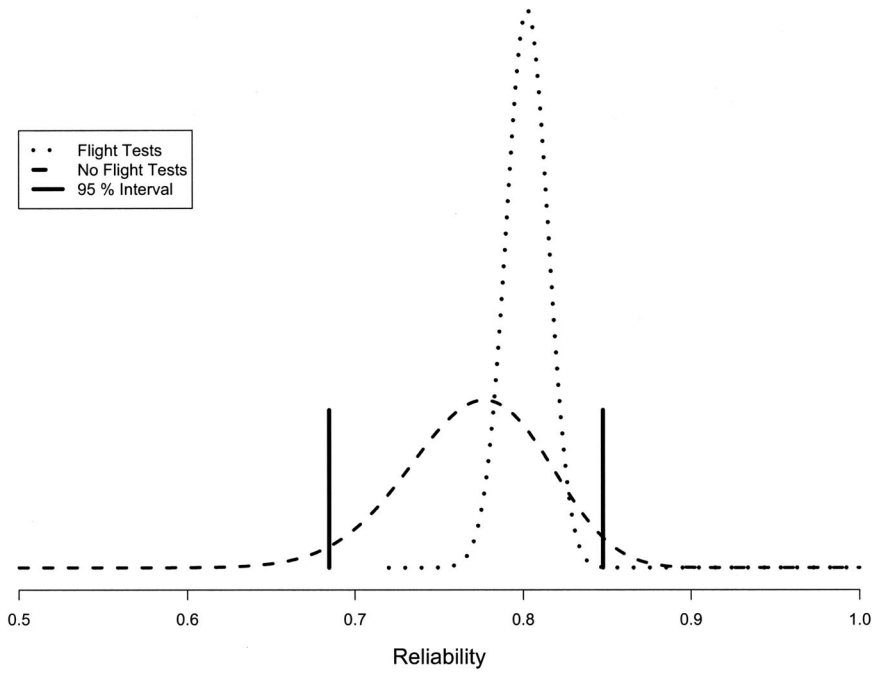

Figure 3. Posterior distributions for the reliability of the system represented in Figure 1. The dotted line is based on the model that included the full-system flight tests; the dashed line shows results without full-system level data. The 95% interval is based on results without full-system level data.

Posterior distributions for all components are given in Figure 4. In the cases where the posterior distributions are very similar, such as C_3, these correspond to cases where component level data are available. When the marginal posterior distribution based on component-level data only is substantially more disperse than the posterior based on results using the flight data, such as C_1, this corresponds to situations where no components level data is available. In these cases, the flight data is highly beneficial since the entire system working coupled with the series form of the system provides information that these components also worked. We note the agreement between the two posterior distributions (full system tests included versus full system tests excluded). In every case, the 95% highest posterior density (HPD) region includes essentially the entire distribution with full system information included. This validates that the full system data and the combined component data are summarizing system reliability similarly, but just with different precision. In addition, it is worth noting that this approach can provide a system estimate of reliability even in the absence of full system data. Note that for this case, since there is a great deal of flight test data, the data dominates the prior in the first calculation.

Also of interest is the posterior distribution for the expert precision parameter $N_{1,2,3}$. The posterior mean for this distribution is 12.2. This suggests that the expert's opinion is worth approximately 12 full system tests. Given the prior mean of 5, we conclude that the expert is reasonably well calibrated with the system structure and data. This is also a useful measure to determine the added benefit of including expert opinion in the analysis.

Diagnostics

Two concerns commonly encountered in modeling system-level reliabilities using series diagrams like that depicted in Figure 1 involve the extent to which different components function independently and whether system (or subsystem) reliability decreases when components are assembled. A simple cross-validation

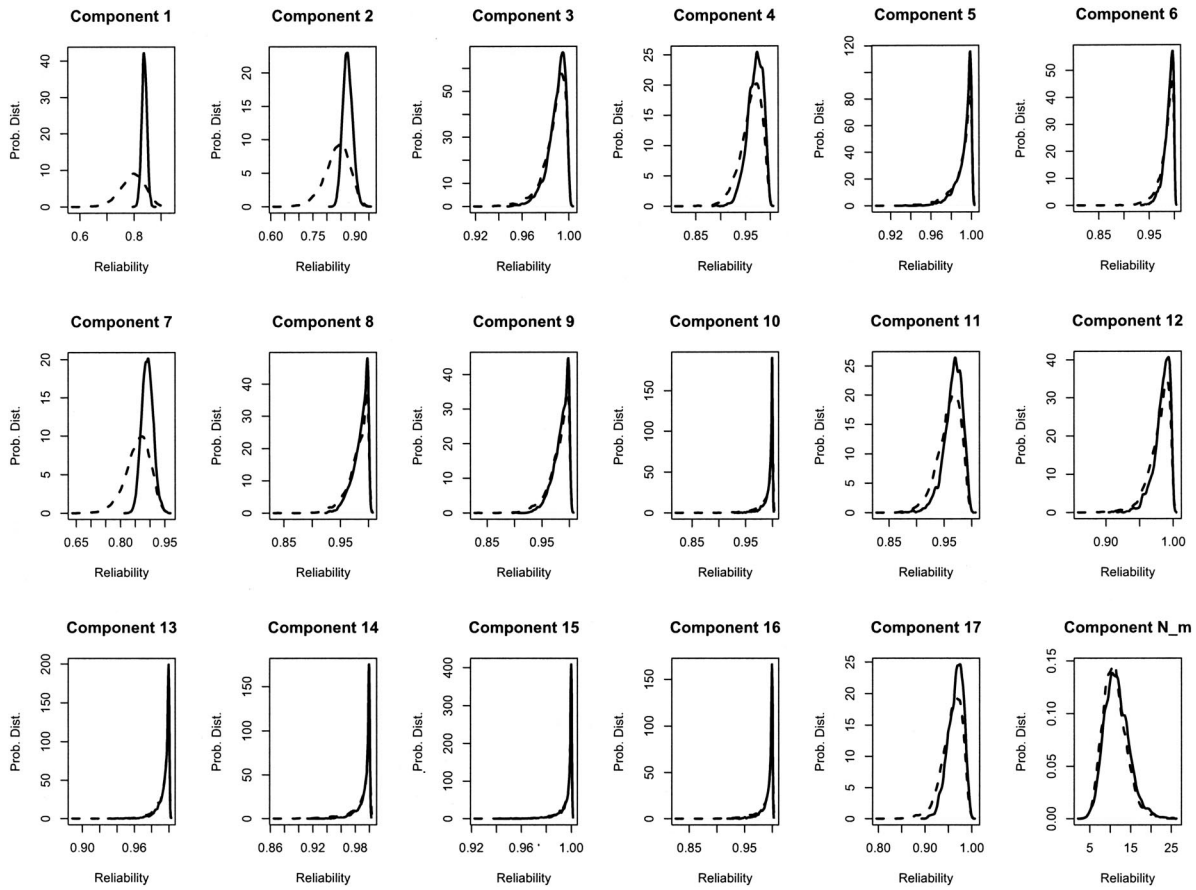

Figure 4. Posterior distributions for the reliability of the system represented in Figure 1. In each pair of plots, the more peaked curve represents the marginal posterior density based on all test data, while the more dispersed curves represent the marginal posterior density using only component-level data (i.e., excluding system-level tests).

diagnostic useful for assessing the importance of these influences can be constructed by iteratively omitting data collected at each node from the estimation procedure, and then examining the predictive distribution for the omitted datum.

Such a procedure is applied to data obtained for this missile system and resulted in an estimate of 0.83 for the predictive probability of observing fewer successes at the system-level than are actually observed. It therefore seems that there is little evidence to support the notion that the reliability of the system is degraded as components are assembled and required to operate as a unit.

There is, however, some indication of model lack-of-fit at the component level. For components 10 and 17, the predictive distribution for observing fewer successes than are obtained at these nodes is approximately 3.5%. The same number of failures are observed at each of these components, and these two components had the highest failure rate of any components in the system. Model lack-of-fit in this instance might thus be attributed to the fact that the hierarchical mean estimated for the terminal nodes, γ, increased substantially when the datum for either of these nodes is omitted, resulting in an overly optimistic estimate of this probability. Possible remedies for such model inadequacy would be to stochastically decrease the prior assigned to the value of J, or to introduce a separate hierarchical group for these nodes. In this case, neither remedy appeared to

substantially affect estimates of system reliability in subsequent sensitivity analyses.

CONCLUSIONS

In this paper, a Bayesian approach to combining component, subsystem and system data with expert judgement is presented. It allows for the flexible combination of multiple sources of data with different weights for the various sources of information. In addition to being computationally manageable with the use of freely available software, the approach reduces the final estimates dependence on which components have data available. This robustness to the structure of available data is appealing, since frequently the availability of data at various components is a function of cost or convenience.

The approach also allows for comparison of results between the complete analysis and an analysis when full system data is not incorporated. The similarity of the two sets of results for this system validate that the system has been appropriately modeled, and provides insights into the usefulness of this approach when full system data may not be available.

ACKNOWLEDGMENTS

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RIST PRIZE
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David Rist Prize: The Rist Prize recognizes the practical benefit sound operations research can have on "real life" decision making and seeks the best implemented military operations research study from those submitted in response to this 2008 Rist Prize Call for Entries. The Call solicits abstracts with letters of endorsement for *implemented* recommendations from studies or other operations research-based efforts, e.g., analyses, methodology improvements, that influenced major decisions or practices. Entries submitted in response to this call will be eligible for consideration for the Rist Prize. There are two cash prizes that may be awarded: $3,000 for first place (i.e. Rist Prize winner) and $1,000 for honorable mention. To be considered:

- An abstract and letter of endorsement for the *implemented* recommendations from the study must be mailed to the MORS office and postmarked no later than **Friday, 16 NOVEMBER 2007**;
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Please send the originals, electronic copies (unclassified abstracts and letters can be emailed to cynthia@mors.org) and a complete MORS Abstract Disclosure Form 109 A/B (available from the MORS Office by telephoning 703-933-9070 or by emailing cynthia@mors.org) to the MORS office at 1703 N. Beauregard St., Suite 450, Alexandria, VA, 22311-1717 (attn: Cynthia L. Kee).

From those entries submitted, the judges will select 3-5 finalists on or about **Friday, 18 JANUARY 2008**.

If selected as a finalist:
- A mentor will be assigned by MORS.
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- The finalists will be invited to present that briefing on the **Monday** prior to the 76th MORSS (**9 JUNE 2008**, at the US Coast Guard Academy, New London, CT). Each finalist will be allotted one (1) hour to present—which includes time for questions and answers. This session will be open to all MORSS attendees. At this session, judges will ask questions as appropriate. Following this session, the judges will select the Rist Prize winner and the honorable mention.

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To be eligible, a presentation must, at a minimum:

- Be an original and self-contained contribution to the systems analysis or operations research of an implementation
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Eligible study presentations are judged according to the following criteria:

Professional Quality
- Problem definition
- Citation of related work
- Description of approach
- Statement of assumptions
- Explanation of methodology
- Analysis of data and sources
- Sensitivity of analyses (where appropriate)
- Logical development of analysis and conclusions
- Summary of presentation and results

Contribution to Military Operations Research and Major Decisions
- Importance of problem
- Contribution to insight or solution of the problem
- Power of generality of the result
- Originality and innovation
- Contribution of the study to the decision

Reducing Aircraft Down for Lack of Parts with Sporadic Demand

Dr. Tovey C. Bachman

LMI
tbachman@lmi.org

ABSTRACT

In the military aerospace environment, certain repair parts are infrequently demanded, but stocked because they are essential to maintaining a weapon system critical to the war-fighter. Because of their sporadic demand, it is difficult to decide when to buy these items and in what quantities. As systems become more reliable and failure rates decrease, the number of these infrequently demanded parts is likely to grow. Earlier studies for the Defense Logistics Agency (DLA) and the Federal Aviation Administration (FAA)—organizations that manage parts inventories for repairing complex systems—found the Peak ordering policy the author invented significantly reduced wholesale wait-time and backorders. Rigorous new experiments confirm the benefits of the Peak Policy™,[a] and show it can reduce retail wait-time and backorders as well. By considering the distribution of retail backorders, or "holes," over an aircraft squadron, we estimate the resulting reduction in the number of aircraft down for lack of parts. We also analyze the policy's near-term effect on inventory value and procurement workload, showing that the Peak Policy can reduce both within a few years of policy initiation. After 7 years of development and review, the Peak Policy is mature enough for implementation. A live test is underway, and broader implementation is under consideration.

BACKGROUND

For frequently demanded parts, there is a well-developed theory and set of processes for ordering that balances the investment in inventory with customer service. Unfortunately, when parts experience only infrequent demand, that theory breaks down, and the established processes no longer work well.

An inventory management system for a single site typically manages each item using two control levels: an item's reorder point (ROP), which determines when to order, and a requisitioning objective (RO), which determines how much to order. An order is placed when assets on-hand plus on-order decrease to or below the ROP, and the difference between the RO and current assets is the quantity ordered. The RO is usually the ROP plus a nominal order quantity, Q, often a Wilson lot size ("economic order quantity"). Thus Q is the quantity ordered if assets drop exactly to the ROP.

The ROP is an estimate of lead-time demand plus a safety level that protects against variability in lead-time demand.

Safety-level computations usually treat the number of demands in a lead-time as a random variable with a tractable theoretical probability distribution (e.g., Poisson, negative binomial, Laplace, or normal), estimate the mean and variance, and derive expressions for expected backorders and inventory cost as a function of the safety level. Mathematical optimization techniques are then used to set item safety levels to balance inventory investment with expected backorders, probability of a stockout, or system availability.

This approach to optimizing ordering policies for a single site began in the 1950s (Galliher et al., 1959), and has developed to include a great variety of policies (Silver, 1998). It also has been extended to optimize policies across a supply chain (Kruse, 1979), account for repair actions as well as ordering actions (Sherbrooke, 1992; Slay et al., 1996), and treat items that apply to diverse weapon systems with distinct availability goals (O'Malley, 1983). When there is sufficient demand data to characterize the lead-time demand distribution, but theoretical distributions do not fit well, non-parametric techniques, such as the bootstrap method (Fricker and Goodhart, 2000), may apply.

But what happens when items experience long and irregular periods of inactivity between demands (6 months to several years), what we call sporadic demand? For these items, the lead-time demand is usually zero. In a previous study of more than 300,000 sporadic-demand items, the author found that nearly 95 percent of lead-time intervals contained no demand. Forecasting lead-time demand for these items is extraordinarily difficult, as is forecasting demand variance. That is why a theoretical demand probability distribution is impractical—mean and variance cannot be estimated in any meaningful way. Use of empirical demand probabilities is possible, but for many sporadic-demand items, the data are too sparse to build a reasonable lead-

APPLICATION AREA:
Logistics, Reliability & Maintainability; Readiness
OR
METHODOLOGIES:
Inventory and Simulation

time demand distribution. For example, if an item's only observed demands in the last 5 years comprise a demand for 8 units and another demand for 50 units, there is no reason to believe a demand for 20 units has a probability of zero. Modern enterprise resource planning (ERP) systems of the commercial sector focus on frequently demanded items; since commercial firms do not typically stock sporadic-demand items, ERPs offer no solution.

Inventory management specialists have long sought a successful approach to setting ROPs and ROs for sporadic-demand items. Usually heuristic policies are employed; and the military services use more-or-less arbitrary levels. Such policies fail to link inventory investment to service level, and generally do not work well. There are more sophisticated approaches. Croston showed that, when there is a constant probability of demand in a time interval, high fill rates can be obtained by basing reorder points on forecasts of both the time of next demand and demand size (Croston, 1972; for a recent survey of articles on policies based on statistical forecast-based methods, see Silver, 1998). Kruse divided an item's population into subsets by pooling items with similar lead-times, prices, and demand frequencies; thereby, obtaining enough demand data for empirical lead-time demand probabilities. Kruse assigned each item subset a common ROP based on a fill rate goal (LMI documented his method earlier; see Bachman and Bosma, 2003). Unfortunately, when demand is as irregular as it is for DLA-managed sporadic-demand items, none of these approaches have been shown to improve service levels (e.g., reduce customer wait-time) without significantly increasing inventory investment.

The scope of the challenge facing DLA—setting cost-effective ROPs for sporadic-demand items—is enormous. The agency manages nearly 1 million aviation stock numbers with sporadic demand, the inventory of those items is valued at more than $1.5 billion and annual sales are in excess of $400 million (Bachman and Bosma, 2003). Although no one item is typically active in any given year, sporadic-demand items experience significant aggregate activity and investment. Furthermore, the lack of DLA parts can render critical weapon systems inoperable—an event with significant military readiness consequences.

Over the last 7 years, the author developed the Peak ordering policy for sporadic-demand items, with the goal of reducing customer wait-times without increasing inventory investment. The most recent work, documented here, was accomplished with the capable support of a number of LMI employees.

OVERVIEW

The focus of this paper is on consumable parts, those deemed uneconomical to repair—when they fail on a weapon system, they are simply replaced. The supply chain for consumable parts is hierarchical, with the weapon system maintainer—the customer—at the end of the chain. To repair an aviation system, the maintainer requests parts from a local supply activity, what we call retail supply. (Retail supply belongs to the military service that owns and maintains the weapon system.) Retail supply requisitions parts from DLA, its wholesale supply organization. In turn, DLA buys parts from its vendors. The section "Aircraft-Level Analysis" illustrates this supply chain.

This paper describes the Peak ordering policy the author developed for DLA's sporadic-demand items and its potential benefits at the wholesale level and the aircraft level, where the lack of a part may ground a weapon system. (While the focus is on aircraft items, there is nothing in this work that suggests the results apply only to aviation). The first two sections describe DLA's current policy—the baseline—and the Peak Policy. The paper then describes the simulation analyses we performed at the wholesale level to compare investment, customer service, and procurement workload for baseline and wholesale policies. The fourth section discusses the near-term effect of converting from DLA's current practice to the Peak ordering policy. The paper then describes simulations of the Peak Policy's effect on part shortages at the aircraft level. The conclusion discusses the status of live testing.

BASELINE AND PEAK POLICIES

Inventory management systems that distinguish between items with more regular demand and those with sporadic demand typically use a three-part policy. One part is an ordering policy for replenishment items (i.e., those replenished regularly) with statistical forecast-based ROPs and ROs. The second part is an activity threshold, typically set in terms of historical requisition frequency and quantity, which separates replenishment items from sporadic-demand items. Part three is a heuristic ordering policy employed for items with activity levels below the threshold.

DLA separates its replenishment items from its sporadic-demand items (which it refers to as numeric stockage objective [NSO] items), with an activity threshold that is based on last year's demand. If an item experiences at least 4 requisitions and at least 12 units demanded in the trailing year,[b] DLA uses a replenishment policy that treats lead-time demand with a Laplace distribution. The agency uses a smoothed forecast of lead-time demand to estimate the mean and a smoothed average of absolute forecast errors to estimate the variance. It also employs a modified Wilson order quantity, and computes each item's expected backorders as a function of the item's order quantity and safety level. DLA then employs Lagrange multiplier optimization to set safety levels across items; thus minimizing the inventory cost required to meet an expected backorder constraint (Presutti and Trepp, 1970).

For NSO items (i.e., items with demand activity below the threshold), DLA uses a heuristic policy that sets the RO to the demand quantity in the trailing year and the ROP to half the RO.[c] This policy does not link inventory cost to service level, however; and special rules apply to small subsets of items (Bachman and Bosma, 2003), which are not considered here. Items also may migrate between replenishment and NSO status quarterly.

The Peak Policy changes the baseline policy in two ways:

- It introduces a new activity threshold between sporadic demand and replenishment, as well as a new way to apply the threshold. Setting this threshold properly is the key to improving service levels with the Peak Policy while controlling inventory value (Bachman and Bosma, 2003).
- It introduces a new ordering policy for the sporadic-demand segment of the item population. In the context of the Peak Policy, we no longer call these NSO items to emphasize the new segmentation.

The new threshold generally results in fewer replenishment items and more sporadic-demand items than the current threshold allows, but there is no change in the ordering policy for replenishment items.

The Peak Policy utilizes quarterly demand data, as does DLA's current policy.[d] We define an item's demand frequency in a time interval as the fraction of quarters with demand, irrespective of quantity. For example, if an item has 2 quarters with demand in a 5-quarter interval, the demand frequency is 0.4—the demand in one of those quarters may be for 1 unit, and in the other it may be for 100 units. The Peak Policy relies on an activity history for each item, which is created from the quarterly demand history that DLA already maintains as follows: Each quarter's demand quantity is replaced by one when that quantity is positive; demand quantities of zero are unchanged. Each quarter, the policy applies a single exponential smoothing forecast (Brown, 1963; Sherbrooke, 1992) to this activity history in order to forecast future demand frequency.

When an item's forecasted demand frequency meets or exceeds the activity threshold, DLA's replenishment policy applies. If the frequency forecast is below the threshold, the new ordering policy applies. This decision is illustrated in Figure 1.

Using an item's quarterly history of demand quantity (DLA maintains at least 10 years' worth), we define its Peak demand as the maximum quarterly demand, in units, in a trailing K-year period. This period is called the look-back (see Figure 2). The look-back may vary by item population, but it is constant across items within a population.

An item's ROP is the product of a price-based multiplier and its Peak demand, where higher multipliers apply to inexpensive items

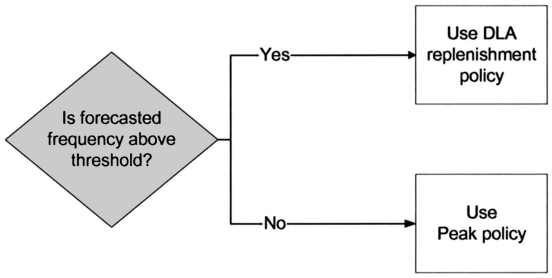

Figure 1. Using Frequency Threshold to Determine Ordering Policy.

and lower multipliers apply to more expensive ones. For example, an item with a unit price of $2.59 might use a multiplier of 5.0, but an item with a unit price of $3,100 might use a multiplier of 0.3—in other words, we can afford a greater level of protection against backorders for the first item than we can for the second. The policy employs a set of price-based order quantities, in which cheaper items receive larger quantities than more expensive items, and sets an item's RO to its ROP plus its order quantity. Larger order quantities for inexpensive items allow us to avoid excessive administrative costs and workload that would result from frequent procurements. Assets on-hand and on-order are compared to the ROP in a continuous review. When assets are at or below the reorder point, the Peak Policy orders enough to bring assets up to the RO.

PEAK POLICY DEVELOPMENT

With the possible exception of its frequency forecast, the Peak Policy may appear simple; however, it is not at all trivial (and it was unclear at the policy's inception whether it was even possible) to find values of control parameters that achieve a high level of service without increasing inventory investment or procurement workload. This paper shows that one can make three-way tradeoffs between customer wait-time, inventory investment, and the number of procurement actions for a wide variety of sporadic-demand item populations. The process is iterative, and is made possible through the use of LMI's Financial and Inventory Simulation Model (FINISIM).

FINISIM provides three modes in which to generate item demand:

- A replay of demand history (retrospective simulation)
- Generation of synthetic demand patterns using empirical demand distributions
- A Poisson distribution.

It models the response of the inventory and related financial systems to demand patterns, emulating a wide variety of operating policies. In particular, FINISIM emulates DLA's current ordering policies, including item migration between NSO and replenishment classification over time and alternative policies tailored for sporadic demand. FINISIM's event processing algorithms are specifically engineered for rapid analyses when events are sparsely distributed over time. In such cases, FINISIM is as much as two orders of magnitude faster than other simulations. This speed is crucial in analyzing numerous alternative policies and finding values of control parameters that achieve a particular performance tradeoff. The model's customer service metrics include average customer wait-time, average backorder duration, number of backorder occurrences, average outstanding backorders, and both unit and requisition fill rates. Financial and workload metrics include annual dollars spent on procurements, number of procurement actions, and average value of on-hand inventory.

We use FINISIM's empirical distribution mode to develop Peak Policies. In this mode the model generates long synthetic demand patterns (e.g., 200 quarters, or 50 years) in which the relative frequency of demands of different sizes, including zero, is close to the frequencies in the actual item demand history. For sporadic-demand items, these demand patterns are too sparse to build a realistic lead-time demand distribution; however, the demand patterns help adjust an ordering policy to respond to a demand pattern with a given frequency of zero demands and a given maximum demand.

There is one limitation of this approach, as far as projecting policy performance is concerned: Each item's generated demands form a stationary stochastic process. That is, there is the same set of probabilities for the number of units demanded at each point in time. A rigor-

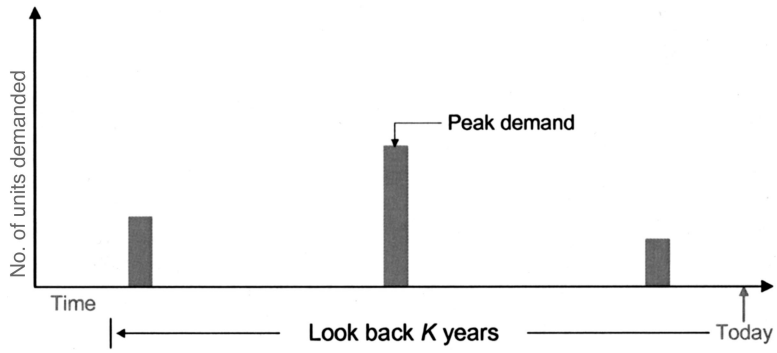

Figure 2. Computing Peak Demand.

ous test of the Peak Policy is discussed later in the section "Wholesale Analysis."

Using FINISIM to test numerous Peak Policies on more than 10 item populations, each with 3,000–15,000 items, led to the conclusion that the best smoothing constant for the frequency forecast is 0.2, and the most cost-effective threshold is 0.6. Although the Peak Policy was originally conceived to apply to items with gaps in demand of a year or more, this result shows the policy actually performs well on items with demand in as many as three out of five quarters. This result appears to be independent of the DLA item population; however, other control parameters must be tuned to the item population to achieve a given three-way performance tradeoff.

Although this process varies with the item population, it generally proceeds as follows:

1) Rank all items by unit price to determine the 25th, 50th, and 75th percentile prices.
2) Assign initial Peak multipliers of 1 for each of the resulting price quartiles, and set all order quantities to 1, as well. Call this policy Peak 1.
3) Use FINISIM to estimate the resulting on-hand inventory value, customer wait-time, and number of procurement actions for the item population.
4) Compare the performance of Peak 1 with FINISIM's assessment of baseline policy.
5) Stop here if the on-hand inventory value from Peak 1 is no more than that of the baseline, the customer wait-time is significantly lower than that of the baseline, and the average number of procurement actions per year is close to the baseline. This usually is not the case; more often, one of the three metrics is not within the desired range (the behavior is dependent upon a combination of item demand patterns and prices.)
6) If the wait-time reduction is not large enough, but the inventory value is lower than the baseline, use a peak multiplier of 2 for the bottom 50th price percentile and leave the top 50th price percentile with a multiplier of 1. If wait-time reduction is significant, but the on-hand inventory value is too high, try a multiplier of 1 for the bottom 50th price percentile and a multiplier of 0.5 for the top 50th price percentile. In either case, call this policy Peak 2.
7) Use FINISIM to compare Peak 2 results with baseline policy. If the metrics are in range, stop; if not, introduce new multipliers (e.g., 4, 2, 1, or 0.5) for all price quartiles.
8) Continue refining the values of multipliers in this way until there is a clear reduction in customer wait-time and the inventory value is less than that of the baseline.
9) With order quantities of 1, procurement actions often exceed the baseline. If so, introduce price-based order quantities, with larger quantities for the lower price quartiles (e.g., 20, 4, 2, or 1). Because this increases on-hand inventory value, reduce the Peak multipliers to counteract the effect.
10) Follow another iterative process to obtain order quantities (and reduced multipliers) that yield procurement actions that are no higher than the baseline, an on-hand inventory value that is no more than the baseline, and a significant reduction in customer wait-time.

To accommodate an especially high demand for the least expensive items, add a bottom 5th percentile price category; if investment is driven heavily by the items with the highest price, add a separate category for items above the 95th price percentile. Although the success of the above process has not been proven mathematically, it has worked for a wide variety of item populations. Peak Policies that result from this process are very efficient in terms of customer wait-time per dollar value of inventory, but we cannot claim they are optimal.

WHOLESALE ANALYSIS

In developing a Peak Policy for an item population, FINISIM projects certain levels of customer wait-time, inventory value, and procurement actions. Because these projections are based on synthetic item demands (as described in the previous section), it was appropriate to construct a rigorous experiment to compare Peak and baseline policies by taking long item-demand histories, developing a Peak Policy based on an initial segment of those histories, and assessing it based on the remaining part of the histories. The demand data used to develop Peak Policies would thus have different demand probabilities from the demands in the subsequent assessment period. If a policy did well, it would not be because of bias in the assessment method.

The author selected populations of DLA-managed items that apply to five critical weapon systems: the AH-64 Apache, E-2C Hawkeye, E-3 Sentry, C-5 Galaxy, and F/A-18 Hornet. We used 9-year quarterly item demand histories for these items, beginning with the first quarter of 1995. In some of the populations, aggregate demand increased over time; in others, it decreased. Items were limited to those with a unique application to each weapon system so it would be clear which weapon system would benefit from any improvement in supply performance. From each population, a sporadic demand subset was extracted, consisting of items that experienced demand in no more than 6 out of the earliest 10 quarters of their demand histories. Each sporadic-demand item population is referred to by the name of the associated weapon system. For example, "C-5 items" designates the sporadic-demand items that apply to the C-5.

For each of the item populations, we used the first 4 years of demand history and FINISIM to develop several Peak Policies, each with a different set of objectives. One Policy, "closest cost match," matched the dollar value of on-hand inventory with that of the baseline policy while decreasing customer wait-time, increasing fill rates, and keeping procurement actions no higher than the baseline. Another Peak Policy, "relax orders constraint," had the same objectives, except it allowed for procurement actions to exceed the baseline. The policy "closest performance match" sought to keep customer wait-time and fill rates close to the baseline while reducing both inventory value and procurement actions. The policy "high performance" had the goal of reducing wait-time increasing fill rates, and allowed inventory value and procurement actions to increase in order to boost service levels. A "compromise" Peak Policy sought to balance improvements in wait-time with reductions in inventory value and procurement actions.

We created a set of Peak Policies that offered a three-way tradeoff between improved service levels (i.e., shorter wait-time and higher fill rates), reduced inventory value, and reduced procurement actions, based on the first 4 years of demand histories. Generally one or two metrics could be improved while constraining a third; however, not every policy option was available across all five populations. As always the ability to develop a particular policy depended upon the joint distribution of item prices and demands in a population.

To assess Peak Policies, we performed retrospective simulations with FINISIM. Each assessment employed the baseline policy from the first quarter of 1995 through the last quarter of 1998—we made no policy change during the development period. We then continued in two ways:

- Start using a Peak Policy at the end of the first 4 years, allow 2 years for simulated procurements to arrive, and then measure performance in the last 3 years.

- Continue using DLA's baseline policy to the end of the 9-year period, measuring performance in the last 3 years.

Figure 3 illustrates our experimental design.

Figure 4 illustrates the results for Peak Policies developed for E-2C items, which was a "middle-of-the-road" case in terms of improvement over the baseline. Each policy name (shown at the top of the chart) refers to the objective used to develop the Peak Policy. The grouped bars show performance for each Peak Policy according to four metrics. Each metric is the percentage difference between the Peak and baseline policies' performance, based on averages over the 3-year assessment period. Lower numbers are better for the first three metrics: wait-time, number of orders placed, and value of on-hand inventory. Higher numbers are better for the fourth metric: unit fill rate. Baseline policy performance, in absolute terms, is shown to the right of each chart.

The first Peak Policy, "closest cost match" produced a nearly 30 percent reduction in wait-time, and a nearly 20 percent reduction in procurement actions, relative to DLA's current policy. Although it was developed to match inventory value with the baseline, the assessment demonstrated an inventory value reduction of approximately 5 percent, which was better than expected.

As expected, the reduction in procurement actions in the second option, "relax orders constraint," was less than that of the first Peak Policy. Allowing more procurement actions than the first Peak Policy resulted in a slightly greater reduction in inventory value than the first policy. Wait-time reduction was still about 25 percent, but it was not as large as that of the first option. So, relaxing the constraint on the number of procurement actions did not produce a significant benefit in the other two metrics relative to the first Peak Policy.

The "high performance" Peak Policy, developed to increase performance significantly, achieved its objective—wait-time was reduced about 45 percent, the number of procurement actions declined, and the value of inventory increased by a little more than 10 percent. Our results indicated a range of Peak Policies—each emphasizing different performance metrics—are available for the E-2C items.

Figures 5 through 8 illustrate the Peak Policy assessment results for the other four weapon systems. The policies shown are only illustrative; many other policies could be developed.

Assessments of item populations generally demonstrate that policies behave in a manner consistent with their objectives.

- Peak Policies developed to reduce wait-time did so in the assessments.
- Peak Policies that sought to reduce procurement actions achieved that goal.
- Policies that attempted to reduce inventory value kept it at or below the value produced by the baseline policy.

For all item populations, we found policies that offered a nice compromise among the competing objectives: reducing wait-time, reducing the number of procurements, and reducing inventory value.

We performed separate analyses to test the threshold condition for item migration. The goal was to determine if there were any negative effects (for example, a large increase in inventory value) if we started with the full item populations (rather than populations with initial demand frequency that did not exceed 0.6) and let the Peak Policy's threshold

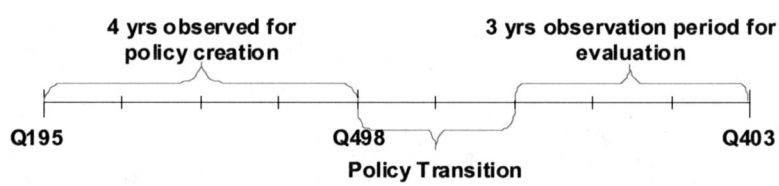

Figure 3. Experimental Design.

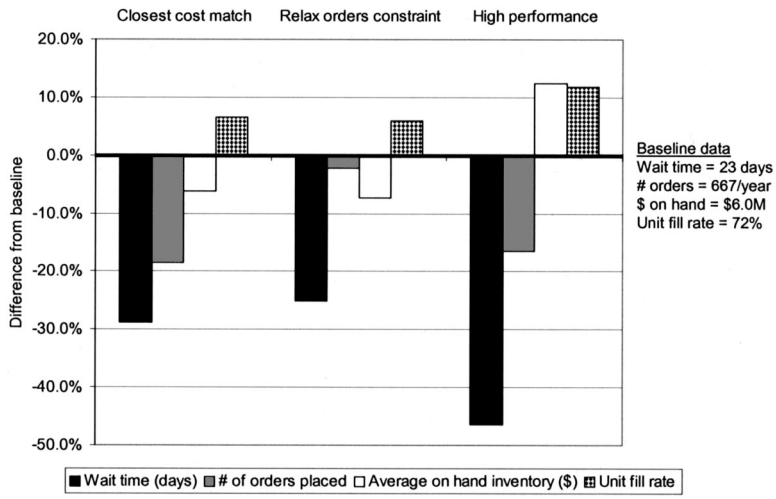

Figure 4. Assessment Results for Peak Policies Developed for E-2C Items.

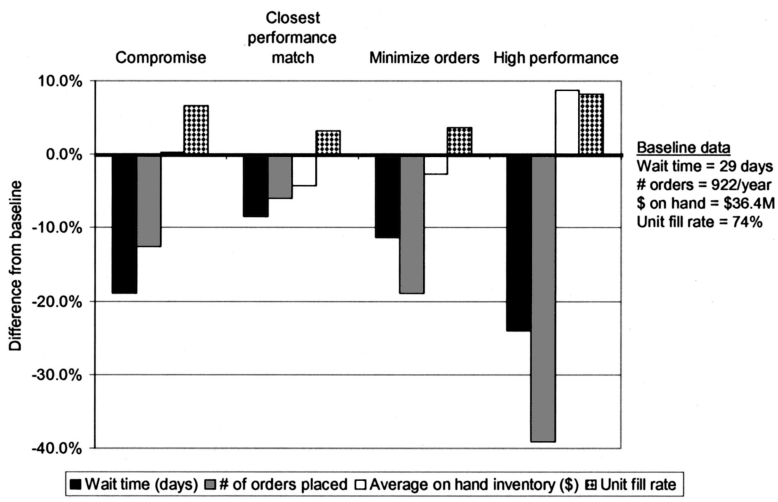

Figure 5. Assessment Results for Peak Policies Developed for AH-64 Items.

control item migration between replenishment and sporadic demand. These separate analyses followed the same timeline for policy conversion and assessment as those shown in Figure 3. We observed no negative effects from allowing the new threshold and the frequency forecast decide when Peak ordering should apply.

Taken together, these results confirm the Peak Policy's benefits. And they show that, for the first time, DLA can make three-way performance tradeoffs for sporadic-demand items.

NEAR-TERM IMPACT

Assessment results in the previous section were 3-year averages, taken over a period that starts after a Peak Policy has been in effect for 2 years. It is natural to ask, "What happens to inventory value and procurement actions when policy conversion occurs?" Short-term increases in these two measures are inevitable when parts are purchased according to a new ordering policy. After all, DLA is purchasing a different mix of items than what the previous policy has "put on the shelf."

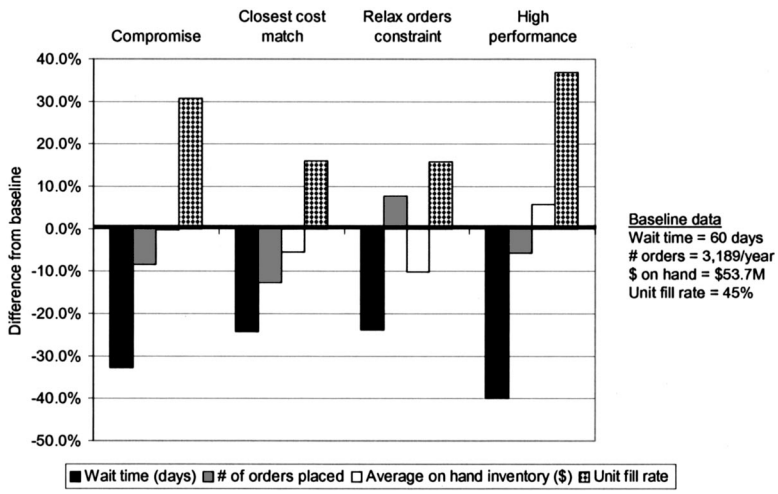

Figure 6. Assessment Results for Peak Policies Developed for C-5 Items.

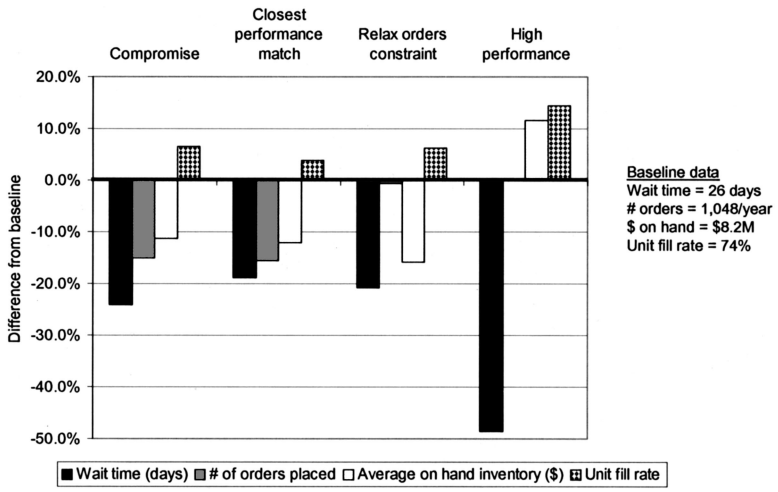

Figure 7. Assessment Results for Peak Policies Developed for E-3 Items.

We used FINISIM to analyze "compromise" policies considered in the previous section, projecting how inventory value and procurement actions change over time. Figures 9 and 10 illustrate the on-hand inventory value and the number of procurement actions by year for the compromise policy developed for C-5 items.

As seen in Figure 9, on-hand inventory was up only slightly over the baseline in year 1. This reflects many procurement lead-times of more than a year. By year 2, inventory value was up significantly, which reflects the arrival of most material ordered at policy conversion. Moving from year 2 to year 3, the value of inventory declined—sales exceeded buys for the first time. This continued as we moved from year 3 to year 4, when inventory value fell below the baseline. Inventory remained below the baseline in year 5, which reflects the purchase of a better mix of spare parts—more of what is bought sells. The C-5 items served as a "middle case." With some weapon systems it took longer for the inventory value to begin declining; with others it began sooner.

Figure 10 illustrates the expected surge of procurement actions in year 1, as the new policy triggered the buys necessary to produce a

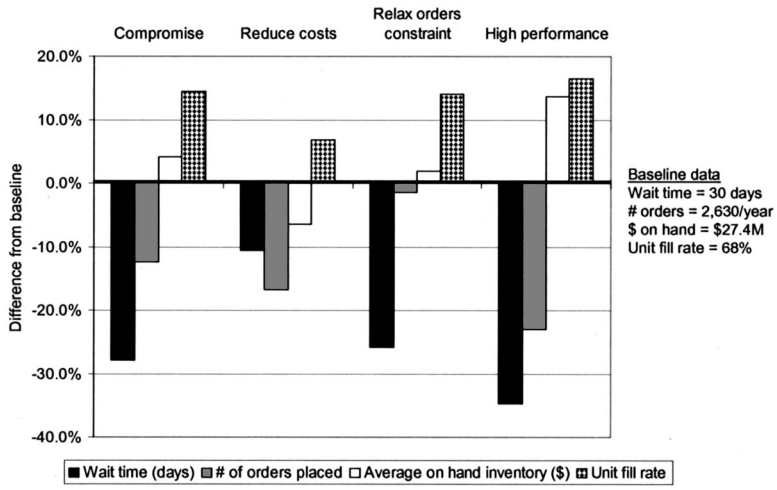

Figure 8. Assessment Results for Peak Policies Developed for F/A-18 Items.

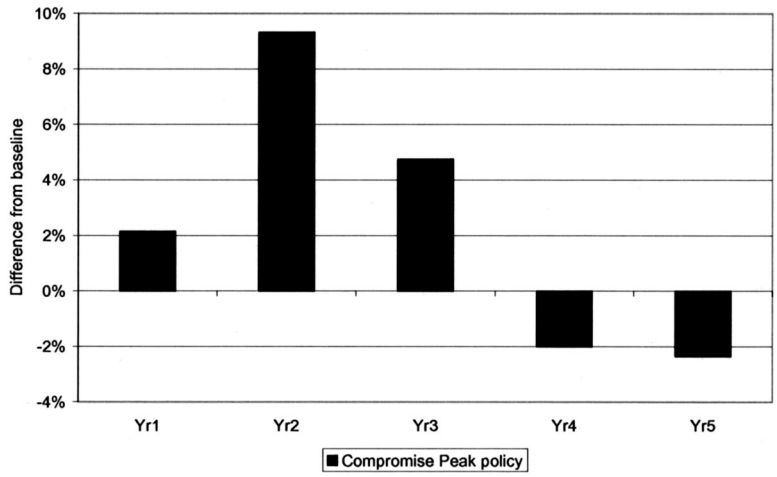

Figure 9. Value of On-Hand Inventory by Year for C-5 Compromise Peak Policy.

new mix of inventory. In year 2 and thereafter, annual procurements were below those of the baseline policy. So, the increase in procurement workload was one-time event, and was consistent across the item populations. Policies developed by LMI for DLA moderate the year-1 procurement workload and spending.

AIRCRAFT-LEVEL ANALYSIS

The wholesale analysis complete, we turned to examining the effect of changing the wholesale ordering policy on parts shortages at the end of the supply chain—the aircraft. An aircraft rendered inoperable ("down") for lack of a part is said to be not mission capable due to supply (NMCS); and the occurrence of a back-order, or "hole," at the aircraft level is referred to as an NMCS incident. We proved that using the Peak Policy at the wholesale level could reduce NMCS incidents.

Because personnel at the aircraft level often resolve an NMCS incident by cannibalizing a part from other weapon systems or finding other workarounds, we could not claim an increase in actual readiness; however, it is DLA's job to make the required parts available, not to assume its customers will work around parts

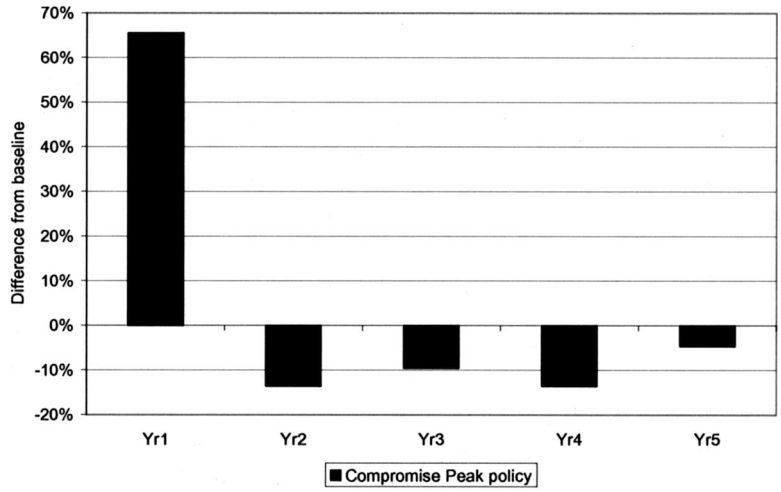

Figure 10. Number of Procurement Actions by Year for C-5 Compromise Peak Policy.

shortages. For this reason, it was appropriate to measure the effectiveness of the Peak Policy through the increase in aircraft availability (the percentage of the fleet not down for lack of a part), ignoring workarounds. We called this "no-cannibalization availability," although other workarounds are discounted as well.

We used populations of DLA items that apply to three types of aircraft: the Navy's E-2C and F/A-18, and the Air Force's E-3. For the first two aircraft, we used items the Navy had identified as first indentures (items removed directly from the aircraft, as opposed to items removed from a reparable component). First indenture items served as proxies for items that could ground the aircraft. For the E-3, we used items that grounded the aircraft at some point in the past. This analysis was not limited to items that had a unique application to the subject aircraft—we included items common to multiple weapon systems. Item populations, and the Peak Policies developed for them, differ from those discussed earlier in "Wholesale Analysis," in which the goal was to show we could produce a three-way performance tradeoff.

From these three item populations, we built wholesale demand histories and extracted sporadic-demand items (i.e., items with demand in no more than 6 out of the first 10 quarters). We then developed Peak Policies for each item population using the first 5 years of quarterly item demand histories (from the beginning of 1995 through end of 1999). The goal of these policies was to significantly reduce wholesale customer wait-time, but keep inventory value and procurement actions near the baseline.

Using a retrospective simulation, we replayed demands for the first 5 years, employing the baseline policy. We activated the Peak Policy at the start of 2000, and allowed it to run for 2 years so that most of the assets from simulated procurements could arrive. We passed each items' simulated wholesale assets (on-hand, due-in from procurement, and backorders) and wholesale levels (ROP and RO) as of the end of 2001 to a multi-echelon supply chain simulation (a version of FINISIM that projects backorders at the aircraft level). Another set of final wholesale assets and levels, which were produced using the baseline policy from the beginning of 1995 to the end of 2001, was also passed to the multi-echelon FINISIM for comparison. Figure 11 illustrates the experimental design.

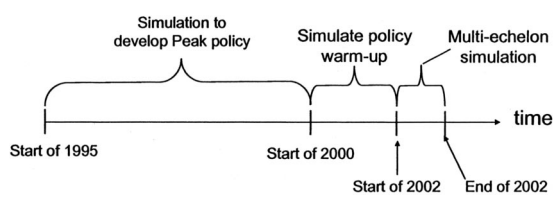

Figure 11. Experimental Design for Aircraft-Level Analysis.

To examine the effect of Peak Policies on DLA's Navy customers, we chose four aircraft carriers, two from the Atlantic Fleet and two from the Pacific Fleet. The carriers and their associated aircraft deckloads (for 2002) were

- USS *George Washington* (4 E-2Cs, 36 F/A-18s),
- USS *John F. Kennedy* (4 E-2Cs, 24 F/A-18s),
- USS *Kitty Hawk* (4 E-2Cs, 36 F/A-18s), and
- USS *John C. Stennis* (4 E-2Cs, 36 F/A-18s).

We performed eight multi-echelon simulations to examine the effect of Peak Policies on the customer, one for each ship-aircraft combination.

For Air Force customers, we considered the three permanent Air Force bases for the E-3:

- Elmendorff AFB
- Kadena AFB
- Tinker AFB.

Of these, only Tinker AFB had significant demand for DLA-managed sporadic-demand items in 2002, so we only performed a multi-echelon simulation for that base. Tinker was assigned 28 E-3 aircraft in 2002.

We analyzed backorders at the aircraft using a daily, multi-echelon simulation for 2002. The lowest echelon of the supply chain was the ultimate customer for parts in our item populations—an aircraft fleet (i.e., one or more squadrons). The next echelon up was the local supply activity, or retail supply. Above that, we modeled wholesale supply, with the top echelon DLA's vendors (i.e., suppliers), which we treated as a single entity. Figure 12 illustrates the simulation for the Navy case.

Simulations for the other aircraft-ship combinations were similar. For the E-3, Tinker AFB replaces the aircraft carrier in Figure 12. At the retail level (Tinker AFB), other customers include demands from depot activities as well as other aircraft based at Tinker AFB. At the DLA level, other DLA customers represent wholesale demands from all locations other than Tinker.

Consider the case of the E-2C squadron on the *John C. Stennis*. We used Navy maintenance data for parts required to replace items removed directly from the E-2C to model requests on the ship's supply activity (from aircraft to aircraft carrier in Figure 12). We also used Navy maintenance data to model demands from other repair activities competing for the same parts (from other on-carrier customers to aircraft carrier in Figure 12), to repair another aircraft type on the same carrier, for example. We emulated the Navy's ordering policy using their levels data; so, when assets were low enough, FINISIM generated a simulated order from the carrier to DLA. We modeled competing wholesale demand, from customers other than the *Stennis*, using DLA requisition history data (from the other DLA customers to DLA in Figure 12.) FINISIM then generated simulated demands for parts from DLA to its vendors. We measured performance at the aircraft, aircraft carrier and DLA levels.

We assessed the effects of Peak Policies on the aircraft in two ways:

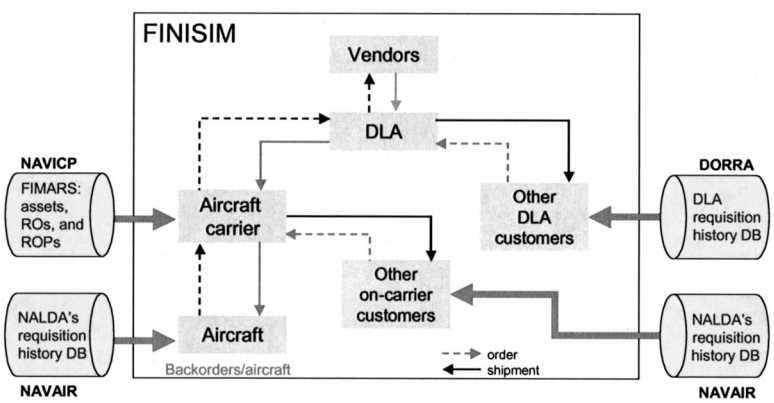

Figure 12. Structure of Multi-Echelon Simulation for Navy Customers.

- We projected the average number of aircraft down for lack of parts at a location by simulating the average number of outstanding backorders at the aircraft level, and considered the resulting holes as uniformly distributed over the aircraft at that location. Any aircraft with at least one hole was down for a part (it was NMCS).
- We used our no-cannibalization availability formula (shown below) to estimate the probability that a randomly chosen aircraft was available at the location.

$$A_0 = \prod_{i=1}^{N}\left(1 - \frac{BO_i}{NAC}\right)$$

where A_0 is availability, the percentage of the fleet not down for a part,

N is the number of parts, BO_i is the number of backorders for part i,

and NAC is the number of aircraft.

Each factor in the product is our estimated probability that an aircraft is not lacking a particular part. Treating holes for different parts independently, the product of all such factors is the probability of an aircraft not being down for any of the subject parts.

For the F/A-18s on two carriers, changing from the baseline to Peak Policy at the wholesale level reduced part shortages and increased no-cannibalization aircraft availability. For the *George Washington*, we reduced average outstanding backorders from 6.35 to 5.36. Distributing the resulting holes uniformly over the 36 aircraft, we had an average of 6 aircraft down with DLA's baseline policy; we had an average of 5 aircraft down with the Peak Policy, and the no-cannibalization availability increased by 3 percent (i.e., the difference between availability with the baseline policy and availability with the Peak Policy, where each is computed using Equation 1). On the *Kennedy*, the Peak Policy reduced the average outstanding backorders from 3.50 to 1.62—two fewer aircraft down due to the lack of a part—and increased availability by 7 percent.

We observed no change in backorders at the aircraft level for F/A-18s on the other two carriers or E-2Cs on any carrier (no positive or negative effects). This was expected; after all, worldwide demand for the parts in question is sparse, even over several years. This means that, at any given customer location, only a few parts are active in a single year. Although the Peak Policy makes parts more available for a population of items, taken as a whole, it does not improve performance for every item—it may not help the few active parts at one customer location. Even if the Peak Policy increases availability for certain parts at the wholesale level, there is no guarantee any particular customer will benefit in a 1-year period—other customers may get the parts first, which leaves no parts for the customer analyzed.

For the Air Force case, the Peak Policy reduced average outstanding E-3 backorders from 29.73 to 27.40—a reduction of two holes. (This excludes one part with extraordinarily large demand in 2002; with this part included, average outstanding backorders would have been reduced from 104 to 85). Using the baseline policy, we had 30 holes on average. Distributing these holes uniformly, all 28 aircraft were down (26 aircraft have 1 hole and 2 aircraft have 2 holes). With the Peak Policy, the average number of holes was 27.4, so we can anticipate either one less aircraft down or no improvement, depending on whether the 27.4 represents 27 holes or 28 holes. In either case we reduced part shortages at the aircraft-level (as we saw in two of the F/A-18 experiments) and no-cannibalization aircraft availability, as defined by Equation 1, increased by 8.6 percent.

From this analysis, we concluded the Peak Policy decreased the number of aircraft down for lack of sporadic-demand DLA parts. In two cases it was a decrease of one or two aircraft; and in a third case, it was one less aircraft down, but only for about half the time. In no case did the Peak Policy increase the number of aircraft down or parts shortages.

We believe the benefit of the Peak Policies is understated, because we did not capture the effect of making DLA-managed parts more available for reparable item repair, which could reduce holes for those items, as well.

Our analysis is significant in another way: To the best of our knowledge, these were the first experiments to model the effects of DLA

wholesale supply policy on supply-oriented readiness measures using realistic customer demand. Modeling demand properly is critical to accurately projecting performance. In previous LMI experiments, wholesale backorders can be too low (by as much as a factor of three) if theoretical demand probabilities, rather than empirical demand data, are used.

LIVE TESTING

In February of 2004, the DLA's Defense Supply Center, Richmond (DSCR), asked LMI to develop a Peak Policy to improve support for 16,000 sporadic-demand items that apply to 15 key aircraft and engines. A joint effort involving DLA and the military services selected specific items for various fixed-wing aircraft (A-10, E-2, E-3, EA-6B, C-5, F-15, F/A-18, and S-3), helicopters (AH-64, CH-47, HH-60G, and UH-60), and engines (F100 series, F404, and TF-39).

The Peak Policy we developed is projected to reduce customer wait-time by 35 percent by year 2 and by 60 percent in the longer term, all while staying within DSCR's guidelines for initial spending and number of procurement actions. Working with DSCR, we produced a variant of the policy that could be implemented within DLA's ordering system, with similar performance and cost parameters.

Starting in March 2004, DSCR began to implement this revised Peak Policy, electing to test it on C-5 items. Because many items have procurement lead-times in the range of 1 to 3 years, we expect results of this test to begin emerging by late summer of 2005.[e] Headquarters DLA is also reviewing the Peak Policy for possibly wider implementation.

CONCLUSION

We developed the Peak ordering policy for items with sporadic-demand patterns, and a simulation model that enables us to make three-way tradeoffs between the resulting service level, value of inventory, and procurement workload. The ability to make tradeoffs—long available for more frequently demanded items—is new for sporadic-demand items. We showed this capability enables us to produce Peak Policies that reduce wholesale customer wait-time by 20 to 45 percent while maintaining or reducing wholesale inventory value and procurement actions. We further demonstrated that the Peak Policy can reduce parts shortages at the weapon system level and reduce the number of aircraft down for lack of a part.

After 7 years of development and review, the Peak Policy is now mature enough for implementation. A live test is underway, and broader implementation is under consideration.

NOTES

[a] Peak Policy is a trademark of Logistics Management Institute.

[b] Since this article was submitted for publication in 2004, DLA has replaced these criteria with a minimum of 4 months out of the last 12 with demand activity and a minimum of 10 units demanded in the last 12 months. Subsequent LMI research has confirmed that the benefits reported in this article are still obtained with the new activity threshold.

[c] DLA now sets the RO to the average annual demand over the last two years and the ROP to half of the RO. Again, we have confirmed that this does not change the benefits reported in this article by performing additional simulation studies.

[d] DLA is now using monthly data; again, simulation studies confirm that the benefits described here are still obtained.

[e] The experiment effectively ended after only a year and a half because most of the subject items had migrated from DLA's legacy system to its new material management system under Business Systems Modernization (BSM). Modest improvement was noted during that limited period, but there was not enough time to see if the full benefit of the Peak Policy could be obtained. DLA is continuing to pursue Peak Policy implementation under BSM.

[f] Now Aging Systems Program

ACKNOWLEDGEMENTS

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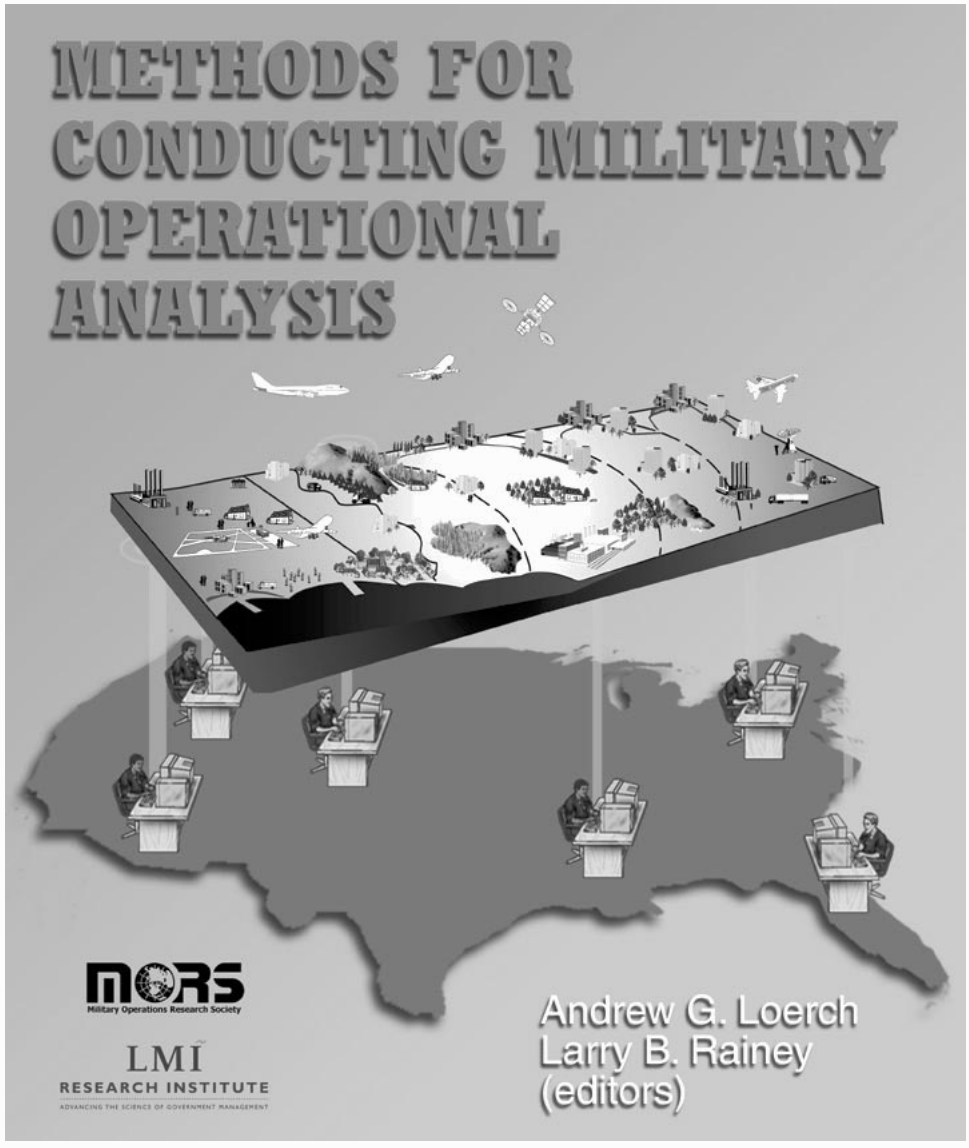

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Two Sensor Motion Models for the Dynamic Sensor Coverage Problem

1Lt Nathan P. Yerrick

59th Test and Evaluation Squadron
nathan.yerrick@nellis.af.mil

Dr. Mesut Yavuz

University of Florida
yavuz@reef.ufl.edu

Dr. David E. Jeffcoat

Air Force Research Laboratory
david.jeffcoat@eglin.af.mil

APPLICATION AREA:
ISR, Unmanned Systems and Air Combat Analysis and Combat ID

OR METHODOLOGIES:
Markov Processes, Stochastic Processes, and Nonlinear Optimization

INTRODUCTION

The allocation of sensors is a significant problem in surveillance, search, and tracking, with applications ranging from sea to space. In this paper, we focus on a class of sensors able to observe multiple areas of interest, one at a time, with a uniform transition time. Sensors with these characteristics could include loitering munitions, undersea or underground robots, or satellites with a focused sensor footprint. The widespread development of low-flying unmanned air vehicles and air-delivered munitions with extended flight times provides the opportunity for these platforms to serve as intelligence, surveillance, and reconnaissance (ISR) assets, pulling information from the battlefield into the global information grid. This paper provides tools for the analysis of these types of systems. The solution methods presented in this paper might be used to give unmanned air vehicles the capability to make autonomous path planning decisions, or to help a mission planner determine the number of target areas that could be covered by a single ISR asset.

We consider a problem in which the objective is for a single sensor to maintain an estimate of a dynamic physical attribute (e.g., position) of multiple targets. This research builds on previous work by Tiwari *et al.* (2005) and Yerrick *et al.* (2006). Tiwari presents a feasibility criterion for a single sensor to maintain a bounded estimate of an attribute at n locations. Yerrick demonstrates by simulation the feasibility criterion presented in Tiwari and develops a heuristic to find a good sensor motion model given the dynamics of the system under observation. This paper provides an optimal sensor coverage solution for two sensor motion models given a model of the observed system's dynamics.

PROBLEM DESCRIPTION

This paper considers a dynamic sensor coverage problem in which there is only one sensor available to cover n sites. Each site is evolving, with site dynamics modeled as a discrete time linear system. In our model, the sensor maintains discrete time Kalman filter estimates of each site, but the sensor can observe only one site at a given instant in time; e.g., the site on which the sensor is focused. A Kalman filter is a set of mathematical equations that provides an efficient computational means to estimate the state of a process, in a way that minimizes the mean of the squared error (Welch and Bishop, 2006). In our paper, the state of the process corresponds to the characteristics of interest at the site under observation by the sensor.

Physical realizations of this problem could include estimating changing temperatures in a building complex or estimating the pollution emission levels of a group of manufacturing plants. In these examples, the states of interest would be temperature and emission levels, respectively. Figure 1 provides an illustration for a three-site scenario. At the time instant pictured, the sensor is focused on site three. In its current position, the sensor can observe the characteristics of site three, but cannot observe sites one or two. In the next discrete time step, we assume that the sensor can move (or refocus) from site three to either of the other two sites, or can maintain its current position. At each time step, the sensor's processor can update its estimate of all the sites. However, the estimates of the unobserved sites are based only on the sensor's model of those sites. At site three, the updated state estimate can be based both on the model of the site's dynamics and the observations, or sensor measurements. The optimal weighting of the model and the measurements of the observed site is a primary function of the Kalman filter.

We assume no cost, or penalty, for sensor movement. The measure of merit is the accuracy of the state estimates maintained for all three sites.

The changing attributes of each site are modeled in the following manner. Consider N independently evolving linear, time-invariant (LTI) systems, whose states x_k are governed by the linear stochastic difference equation

$$x_{k+1} = Ax_k + w_k \tag{1}$$

with a measurement y_k given by

$$y_k = Cx_k + v_k. \tag{2}$$

Matrix A in equation (1) models the dynamics of the system by relating the state at time k to the state at time $k + 1$. Matrix C in the measurement equation (2) relates the state x_k to the measurement y_k. The random

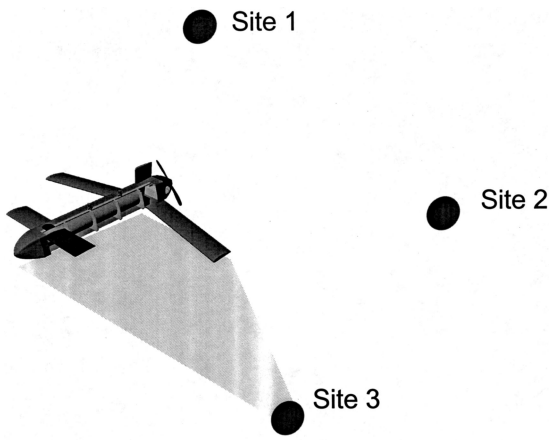

Figure 1. Three Site Example

variables w_k and v_k represent process and measurement noise, respectively. We assume w_k and v_k are statistically independent of one another and that w_k and v_k are both Gaussian random vectors with zero mean and process noise covariance Q and measurement noise covariance R, respectively. We assume that A, C, Q and R are time-invariant. In this paper, we track only one attribute per site. Since all sites are assumed statistically independent of other sites, all the information related to the observed system can be found on the diagonals of A, C, Q, and R. For instance, all information regarding the dynamics of site two would be contained at A_{22} and Q_{22}. Here we follow the notation of Welch and Bishop (2006) for Kalman filters.

As stated earlier, a Kalman filter is a method to estimate the state of a process. In our paper, the state of the process corresponds to the characteristics of interest at the site under observation by the sensor. We assume we're given models of the process and the sensor. Our task is to construct a sensor motion plan, and then to evaluate that plan with respect to the quality of the resulting state estimates.

The sensor's position as a function of time can be modeled as an independently and identically distributed (IID) random process or as an ergodic discrete-time discrete-state Markov chain. The ergodic property ensures the probability distribution of the sensor location always converges to a limiting stationary distribution, no matter the initial condition of sensor location (Kleinrock, 1975). We will show that both the IID model and the Markov chain model have unique advantages. The IID model requires fewer calculations to get a solution, but there are problems (illustrated in the six-site example below) that cannot be solved with an IID sensor motion model, but can be solved with the increased flexibility of the Markov chain model. On the other hand, if the dynamics of one site dominate the problem, so that the sensor would need to spend more than one-half its time at a single site, then the Markov Chain sensor motion model cannot be used, at least with the method presented here. The IID model has no such restriction.

We assume that the sensor can move from one site to another in one time iteration, can make instantaneous measurements, and that there is no cost associated with movement or measurement. The Markov chain has transition probability matrix T, where T_{ij} is the probability that the sensor will move to site j at time $k + 1$ given that the sensor is at site i at time k. The ith diagonal entry of the transition probability matrix, T_{ii}, denotes the probability that the sensor remains at site i at time $k + 1$ given that the sensor is at site i at time k. We denote by π_i the steady-state probability of finding the sensor at site i. We assume that the sensor is focused on exactly one site at each time step, so that $\Sigma_i \pi_i = 1$.

Tiwari *et al.* (2005) provides conditions to determine whether the coverage problem is solved for both the IID and Markov sensor motion models.

1) For the IID case, the coverage problem is solved if

$$\sum_{i=1}^{n} \frac{1}{\alpha_i^2} \geq n - 1 \tag{3}$$

Variable n is the number of sites and α_i is the eigenvalue of A associated with site i. Under the assumption of statistically independent sites, α_i is simply the diagonal element A_{ii}.

2) For the Markov case, the sensor solves the coverage problem if all of the following conditions hold:

$$\frac{1 - \pi_i(2 - T_{ii})}{1 - \pi_i} < \frac{1}{\alpha_i^2},\ i \in \{1, 2, \ldots, n\} \tag{4}$$

Note that the right hand side of (4) is constant since site dynamics are known. The left hand side, however, can be determined only after the transition probability matrix is constructed. Note also that the states of an ergodic Markov chain are recurrent, so that there are no absorbing states and thus no $\pi_i = 1$; see Kleinrock (1975) for definitions of these terms.

Yerrick *et al.* (2006) uses Tiwari's feasibility inequality (4) to construct a performance indicator (PI_i) for each site i:

$$PI_i = \alpha_i^2\left(\frac{1 - \pi_i(2 - T_{ii})}{1 - \pi_i}\right),\ i \in \{1, 2, \ldots, n\} \tag{5}$$

The performance indicator is constructed so that the smaller PI_i, the more effective the observation of site i. If $PI_i < 1$, site i is said to be stable; otherwise, it is unstable. The term "unstable" here means that the site is changing so fast that the sensor's processor cannot maintain an estimate of the conditions at that site. In the vocabulary of the Kalman filter, the difference (as a function of time) between the estimated state and the true state is unbounded. Stated another way, the present sensor motion plan (described by π and T), cannot solve the problem.

Noting that $1 < 2 - T_{ii} \leq 2$ for all T_{ii}, and ignoring the relationship between π_i and T_{ii}, we obtain the following:

$$PI_i = \alpha_i^2\left(\frac{1 - k\pi_i}{1 - \pi_i}\right),$$
$$i \in \{1, 2, \ldots, n\},\ k \in \{1, 2\} \tag{6}$$

This simplification makes it clear that increasing π_i decreases PI_i, which agrees with the intuitive notion that increasing the sensor time allocated to a site increases the likelihood that the site will be stable. Clearly, increasing sensor coverage at one site reduces the coverage time available for other sites. Since our purpose is to construct a transition probability matrix that makes all the sites stable, we define the following objective function:

$$\text{minimize}\left\{\max_{i\in\{1,2,\ldots,n\}}\left(\alpha_i^2\left(\frac{1 - \pi_i(2 - T_{ii})}{1 - \pi_i}\right)\right)\right\} \tag{7}$$

In the section that follows, we develop methods to construct sensor motion strategies that are optimal with respect to this objective function.

OPTIMAL SOLUTIONS FOR THE SENSOR MOTION MODEL

Case I: Sensor Motion Independently and Identically Distributed

The IID case can be modeled using the Markov chain framework; e.g., for a three-site problem,

$$T = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_1 & \pi_2 & \pi_3 \\ \pi_1 & \pi_2 & \pi_3 \end{bmatrix}. \tag{8}$$

Since in the IID case $T_{ii} = \pi_i$, the objective function (7) can be rewritten as

$$\min\left\{\max_{i\in\{1,2,\ldots,n\}}\left(\frac{\alpha_i^2(1 - \pi_i(2 - \pi_i))}{1 - \pi_i}\right)\right\}$$
$$= \min\{\max_{i=\{1,2,\ldots,n\}}(\alpha_i^2(1 - \pi_i))\} \tag{9}$$

The performance indicator for site $i \in \{1, 2, \ldots, n\}$ can be changed to

$$PI_{\mathrm{i}} = \alpha_i^2(1 - \pi_i). \tag{10}$$

Based on the observation that the maximum of the objective function will be minimized when the performance indicators for all sites are equal, we set the ith performance indicator equal to the nth performance indicator:

$$\alpha_i^2(1 - \pi_i) = \alpha_n^2(1 - \pi_n). \tag{11}$$

This leads to the relationship

$$\frac{(1 - \pi_i)}{(1 - \pi_n)} = \left(\frac{\alpha_n}{\alpha_i}\right)^2. \tag{12}$$

Summing both sides of equation (12) and solving for π_n gives:

$$\pi_n = 1 + \frac{(1-n)}{\sum_{i=1}^{n}\left(\frac{\alpha_n}{\alpha_i}\right)^2}. \tag{13}$$

Solving equation (12) for π_i and using the value of π_n from equation (13) provides the remaining values of π, as shown in equation (14).

$$\pi_i = 1 + (\pi_n - 1)\left(\frac{\alpha_n}{\alpha_i}\right)^2, \quad i = \{1, 2, \ldots, n-1\}. \tag{14}$$

We now have the steady-state fraction of time that the sensor should spend at each site. For the IID case, π also contains the optimal sensor motion strategy: at each time step, no matter the current location, the sensor should move to site i with probability π_i. This strategy will ensure the steady-state coverage that is optimal with respect to our objective function.

Case II: Sensor Motion Modeled as a Markov Chain

a. Finding the Appropriate Equilibrium Probabilities from the System Dynamics

When our sensor motion is modeled as a Markov chain, we have two tasks. First, to find the optimal fraction of time to spend at each site (π), and then to find a transition probability matrix T that will lead to the optimal steady-state coverage. This sensor motion model is more flexible, because we can consider the sensor's current location in the sensor movement strategy. However, the analysis is more complex. We begin with the assumption that system dynamics do not vary dramatically from site to site, so that no one site dominates the sensor's attention. Specifically, we assume that no element of π is greater than one-half. In this scenario, it's reasonable to assume that the diagonal elements of T are zero, so that the sensor will never stay at one site for two consecutive time steps. In other words, once a current measurement is obtained at a site, it will always be advantageous to obtain a measurement at a different site, especially since there's no cost associated with sensor movement.

With the diagonal elements of T set to zero, the objective function (7) can be simplified.

$$\min\left\{\max_{i\in\{1,2,\ldots,N\}}\left(\frac{\alpha_i^2(1-2\pi_i)}{1-\pi_i}\right)\right\}. \tag{15}$$

The performance indicator for site $i \in \{1, 2, \ldots, n\}$ becomes

$$PI_i = \left(\frac{\alpha_i^2(1-2\pi_i)}{1-\pi_i}\right). \tag{16}$$

Again reasoning that the maximum of the objective function is minimized when the performance indicators for all n sites are equal, we set the performance indicator for site i equal to the performance indicator for site n.

$$\left(\frac{\alpha_i^2(1-2\pi_i)}{1-\pi_i}\right) = \left(\frac{\alpha_n^2(1-2\pi_n)}{1-\pi_n}\right) \tag{17}$$

Then, solve for π_i for the first n − 1 site locations:

$$\pi_i = 1 - \frac{\alpha_i^2}{2(\alpha_i^2 - \alpha_n^2) + \frac{\alpha_n^2}{1-\pi_n}}, \quad \forall\, i = \{1, 2, \ldots, n-1\}. \tag{18}$$

Using equation (18), we obtain an expression for the sum over all π_i

$$\sum_{i=1}^{n}\pi_i = \sum_{i=1}^{n-1}\left(1 - \frac{\alpha_i^2}{2(\alpha_i^2 - \alpha_n^2) + \frac{\alpha_n^2}{1-\pi_n}}\right) + \pi_n. \tag{19}$$

We also know that $\Sigma_{i=1}^{n}\pi_i = 1$. Therefore, we have

$$\sum_{i=1}^{n-1}\left(1 - \frac{\alpha_i^2}{2(\alpha_i^2 - \alpha_n^2) + \frac{\alpha_n^2}{1-\pi_n}}\right) + \pi_n = 1, \tag{20}$$

a single equation with a single unknown (π_n). However, equation (20) is a complex function of π_n, thus not directly solvable; i.e. π_n cannot be directly derived from this equation. To determine π, we propose the following procedure

(essentially a structured trial-and-error approach).

First, assign an arbitrary value to π_n, such that $\pi_n = p$, $0 < p \leq 0.5$. Note from equation (19) that $\Sigma_{i=1}^n \pi_i$ is a monotonically increasing function of π_n, thus we can systematically increase p if $\Sigma_{i=1}^n \pi_1 < 1$ or decrease p if $\Sigma_{i=1}^n \pi_i > 1$. We use the following equation for steps 1–3 below, which is equation (18) with π_i and π_n replaced by θ_i and θ_n, respectively.

$$\theta_i = 1 - \frac{\alpha_i^2}{2(\alpha_i^2 - \alpha_n^2) + \dfrac{\alpha_n^2}{1 - \theta_n}}, \quad \forall\, i = \{1, 2, \ldots, n-1\} \tag{21}$$

ALGORITHM 1: Obtain the π vector

Step 1: Assign $\theta_n = \rho$, where $0 < \rho \leq 0.5$.

Step 2: Solve for θ_i, $i = \{1, 2, \ldots, n-1\}$.

Step 3: If $\Sigma_{i=1}^n \theta_i < 1$, then $\pi_n = p = 1.1\,\rho$. Else, $\pi_n = p = 0.9\,\rho$.

Step 4: Solve for π_i, $i = \{1, 2, \ldots, n-1\}$, using equation (18).

Step 5: Define $k = \dfrac{(p - \rho)}{\Sigma_{i=1}^n \pi_i - \Sigma_{i=1}^n \theta_i}$.

Step 6: While: $|\Sigma_{i=1}^n \pi_i - 1| > \varepsilon$, $(\varepsilon \ll 1)$

Execute:

1) Set $\pi_n = p$.
2) Solve for π_i, $i = \{1, 2, \ldots, n-1\}$, using equation (18).
3) Set $p = p + k\,(1 - \Sigma_{i=1}^n \pi_i)$

b. Finding an Appropriate T from the Equilibrium Probabilities

Using the computed equilibrium probabilities, here denoted $\pi_{desired}$, the next step is to find a transition probability matrix that will yield these steady-state probabilities. The relationship between the equilibrium probabilities and the transition probability matrix, which can be found in Kleinrock (1975), is

$$\boldsymbol{\pi} = \boldsymbol{\pi T} \tag{22}$$

Even with the diagonal elements of T set to zero, there are still more unknown elements in T than linearly independent equations available to solve for the elements of T, so that there exists an infinite number of T matrices that will satisfy equation (22). Our goal is to find one.

First, we consider the three-site case, and then generalize our findings to the n-site case. The assumption that the diagonal elements are zero is the key to the derivation of the transition probability matrix for the three-site case. First, define T as in equation (23):

$$T = \begin{bmatrix} 0 & T_{12} & 1 - T_{12} \\ T_{21} & 0 & 1 - T_{21} \\ T_{31} & 1 - T_{31} & 0 \end{bmatrix}. \tag{23}$$

It can be shown that there exist elements of matrix T, $0 \leq T_{12}, T_{21}, T_{31} \leq 1$, such that the associated Markov chain is both irreducible and aperiodic (proof omitted). It follows that the convergent probabilities can be calculated using the following set of equations, obtained by expanding equation (22).

$$T_{21}\pi_2 + T_{31}\pi_3 = \pi_1 \tag{24}$$

$$T_{12}\pi_1 + (1 - T_{31})\pi_3 = \pi_2 \tag{25}$$

$$(1 - T_{12})\pi_1 + (1 - T_{21})\pi_2 = \pi_3 \tag{26}$$

Equations (24–26) represent a linear system for which an infinite number of solutions exist, since there are three unknowns (T_{12}, T_{21}, T_{31}) and only two linearly independent equations. Therefore, we need to fix one of the variables (T_{12}, T_{21}, or T_{31}) before we can solve the system. We choose to fix T_{12}. Using equations (25–26), we derive the following relationships.

$$T_{12} = \frac{\pi_2 + (T_{31} - 1)\pi_3}{\pi_1} \tag{27}$$

$$T_{12} = \frac{\pi_1 + (1 - T_{21})\pi_2 - \pi_3}{\pi_1} \tag{28}$$

Note that equation (27) is a linear increasing function of T_{31} and equation (28) is a linear decreasing function of T_{21}. The definition of (transition) probabilities implies that $0 \leq T_{12}$, T_{21}, $T_{31} \leq 1$. Combining all these relationships allows us to derive the upper- and lower-bounds given in Table 1 for the feasible values of T_{12}.

Without loss of generality, assume $0 < \pi_1 \leq \pi_2 \leq \pi_3 < 1$. Then $\pi_2 - \pi_3/\pi_1 < 0$ and $\pi_1 - \pi_3/\pi_1 < 0$, making the lower-bound 0. For the upper-bounds, we see that $\pi_2/\pi_1 > 1$, and hence, is redundant. The last upper-bound, in contrast, is not redundant, since U

Table 1. Bounds for the feasible values of T_{12}.

Lower Bound	Upper Bound	Source
0	1	Definition
$\frac{\pi_2 - \pi_3}{\pi_1}$	$\frac{\pi_2}{\pi_1}$	(27)
$\frac{\pi_1 - \pi_3}{\pi_1}$	$\frac{\pi_1 + \pi_2 - \pi_3}{\pi_1}$	(28)

$= \frac{\pi_1 + \pi_2 - \pi_3}{\pi_1} \leq 1$. Next, we further investigate U to define the feasible region of T_{12}.

We've already assumed that no element of π is greater than one-half. However, for completeness, note that if $\pi_3 > 0.5$, then $U < 0$. The proof is trivial as $\pi_3 > 0.5$, so $\pi_3 > \pi_1 + \pi_2$. Otherwise, i.e., $\pi_3 \leq 0.5$, the feasible region of T_{12} is a non-empty interval, with $T_{12} \in (0, U \leq 1]$.

Consequently, given the optimal limiting probabilities $0 < \pi_1 \leq \pi_2 \leq \pi_3 < 0.5$, we can calculate U and then set $T_{12} = kU$, for some $0 < k \leq 1$. Using equations (25–26), the remainder of the transition probability matrix can be populated easily.

In the three-site case there are three independent variables (T_{12}, T_{21}, T_{31}), but in general the number of independent variables is $n(n - 2)$. This means that instead of having to fix one variable as in the three-site case, $n^2 - 3n + 1$ of the independent variables must be fixed in the n-site case, which makes a mathematical solution problematic. Therefore, a computational method is used for the n-site case.

Here we present a brief outline of the algorithm for the n-site case, with details provided in the MATLAB® code in the appendix. Using MATLAB® V7.0 on a laptop computer with a 1.73 GHz processor, calculating the desired equilibrium probabilities and finding a suitable transition probability matrix for fifteen sites took an average of 0.1 seconds. Both the objective function values and the run times for the method presented here are superior to results obtained in our previous research using a scatter search heuristic for similar cases (Yerrick *et al.* 2006).

ALGORITHM 2: Populate the T matrix

Step 1: Use ALGORITHM 1 to calculate the equilibrium probabilities, π. Let each row in T be the equilibrium probabilities, as in equation (8).

Step 2: Set each element on the diagonal of T to zero, and normalize each row so that every row sum is one.

Step 3: Find the equilibrium probabilities ($\pi_{calculated}$) associated with the current T. $\pi_{calculated} = \pi_{desired}\, T$, where $\pi_{desired}$ are the equilibrium probabilities derived from the system dynamics using equations (13–14).

Step 4: The desired equilibrium probabilities ($\pi_{desired}$) are subtracted from the calculated probabilities: $d = \pi_{calculated} - \pi_{desired}$ [d is a $(1 \times n)$ vector]

Step 5: Update matrix T. Each row of T is updated by subtracting row vector d.

Step 6: The process is repeated from *Step 2* until the sum of the absolute value of the elements of d is less than some predetermined threshold.

The solutions for both the three-site and n-site cases for the Markov sensor motion model are based on the assumption that system dynamics do not vary dramatically from site to site, so that no site requires more than one-half the sensor's attention. We specifically assumed that $\pi_n \leq 0.5$ (recall that we can always number the sites so that site n has the fastest dynamics) and that the diagonal elements of the transition matrix are zero. If the dynamics at site n dominate so that $\pi_n > 0.5$, then the Markov sensor motion model cannot be used, at least with the method presented here. The development of methods for this case is the subject of further study. If the sensor's position as a function of time is modeled as an independently and identically distributed random process, then the methods presented in this paper can be used with no limitation on the stationary probabilities.

EXAMPLES

Two examples are provided: a three-site case and a six-site case. Solutions are provided for both cases. The three-site case illustrates that the IID model requires fewer calculations than the Markov chain model. However, the three-site case also shows that the increased flexibility of the Markov chain model can provide a superior solution. The six-site case pro-

vides an example for which no IID sensor motion solution exists; the Markov chain strategy is required to obtain a solution.

Example 1

In this three-site example, we have the following scenario: a single unmanned aerial vehicle (UAV) is tasked to track the emission levels of a certain chemical from three exhaust outlets in a manufacturing complex. The exhaust outlets are physically positioned in the form of an equilateral triangle, so that UAV can travel between any two outlets in one time step, and the measurement of the emission level is essentially instantaneous as the UAV flies directly over the exhaust outlet. Based on studies of similar manufacturing processes, we assume that the dynamics of the three sites evolve according to the model:

$$A = \begin{bmatrix} 1.10 & 0 & 0 \\ 0 & 1.20 & 0 \\ 0 & 0 & 1.30 \end{bmatrix}.$$

1: IID method: the equilibrium probabilities that minimize the objective function in (9) are obtained by first using Equation (13) to calculate

$$\begin{aligned} \pi_3 &= 1.00 + (1.00 - 3.00)/((1.30/1.10)^2 \\ &\quad + (1.30/1.20)^2 + (1.30/1.30)^2) \\ &= 1.00 + (-2.00)/(1.40 + 1.17 + 1.00) \\ &= 1.00 - 2.00/3.57 = 1.00 - 0.560 \\ &= 0.440. \end{aligned}$$

This result is then used in Eq. (14) to obtain

$$\pi_1 = 1.00 + (0.440 - 1)(1.30/1.10)^2 = 0.218 \text{ and}$$

$$\pi_2 = 1.00 + (0.440 - 1)(1.30/1.20)^2 = 0.342.$$

We now have $\pi = [0.218\ 0.342\ 0.440]$. The associated objective function value is calculated using (9),

$$\max\,(1.10^2(1.00 - 0.218), 1.20^2(1.00 - 0.342), 1.30^2(1.00 - 0.440) = \max\,(0.947, 0.947, 0.947) = 0.947.$$

For this problem, the optimal IID strategy is as follows: at each time step, no matter the current location, the sensor should move to site one with probability 0.218, to site two with probability 0.342, or to site three with probability 0.440. This strategy will ensure the steady-state coverage that is optimal with respect to our objective function.

2: Markov method: the equilibrium probabilities that minimize the objective function in (15) are obtained using Algorithm 1. To get started, we select $\rho = 0.500$ and $\varepsilon = 0.001$. Note that the user has flexibility in selecting these values.

Step 1: set $\theta_3 = 0.500$.
Step 2: using (18), we obtain $\theta_1 = 0.500$ and $\theta_2 = 0.500$.
Step 3: since the summation is greater than 1, we set $\pi_3 = p = 0.9\ \rho = 0.450$.
Step 4: using (18), we obtain $\pi_1 = 0.427$ and $\pi_2 = 0.440$.
Step 5: $k = (0.450 - 0.500)/((0.427 + 0.440 + 0.450) - (0.500 + 0.500 + 0.500)) = 0.274$.
Step 6: perform until the π vector converges.

The remaining iterations are summarized in Table 2.

As shown in Table 2, the resulting equilibrium probability vector is given by $\pi = [0.294\ 0.338\ 0.368]$. A transition probability matrix leading to this steady-state coverage can be obtained by using Algorithm 2. Once again, we select an error threshold $\varepsilon = 0.001$. In Step 1, we initiate a matrix using the equilibrium probabilities found above. Steps 2–6 are performed iteratively until the resulting error is less than

Table 2. Calculations with Algorithm 1

Iteration	π_1	π_2	π_3	$\Sigma_{i=1}^{3}\ \pi_i$
1	0.427	0.440	0.450	1.318
2	0.285	0.331	0.363	0.979
3	0.295	0.338	0.369	1.002
4	0.294	0.338	0.368	1.000

the selected threshold. The calculations are shown in Table 3.

The resulting transition probability matrix is

$$T = \begin{bmatrix} 0 & 0.449 & 0.551 \\ 0.391 & 0 & 0.609 \\ 0.440 & 0.560 & 0 \end{bmatrix},$$

which yields an objective function value of 0.706, a 25% reduction from the objective value obtained by the IID method. Although the IID method requires fewer calculations, it appears that the increased flexibility inherent in the Markov sensor motion strategy allows superior coverage of the three sites. Note that the optimal Markov strategy depends on the current sensor location. For example, if the sensor is currently focused on site one, it should move to site two at the next time step with probability 0.449, or to site three with probability 0.551. If, however, the sensor is focused on site two, it should move to site one with probability 0.391, or to site three with probability 0.609.

Example 2

In this six-site example, the scenario is as follows: a satellite can view six locations of a region by redirecting its sensor while maintain-

Table 3. Calculations with Algorithm 2

Iter.	Site	T (raw)			T (normalized)			$\pi_{desired}$	$\pi_{calculated}$	d	Error
1	1	0.000	0.338	0.368	0.000	0.478	0.522	0.294	0.321	0.027	0.055
	2	0.294	0.000	0.368	0.444	0.000	0.556	0.338	0.337	−0.001	
	3	0.294	0.338	0.000	0.466	0.534	0.000	0.368	0.341	−0.027	
2	1	0.000	0.479	0.548	0.000	0.466	0.534	0.294	0.307	0.013	0.028
	2	0.417	0.000	0.583	0.417	0.000	0.583	0.338	0.339	0.001	
	3	0.438	0.535	0.000	0.450	0.550	0.000	0.368	0.354	−0.014	
3	1	0.000	0.465	0.548	0.000	0.459	0.541	0.294	0.300	0.006	0.015
	2	0.404	0.000	0.597	0.404	0.000	0.596	0.338	0.340	0.002	
	3	0.438	0.548	0.000	0.444	0.556	0.000	0.368	0.361	−0.007	
4	1	0.000	0.457	0.548	0.000	0.455	0.545	0.294	0.297	0.003	0.008
	2	0.398	0.000	0.604	0.397	0.000	0.603	0.338	0.339	0.001	
	3	0.438	0.555	0.000	0.441	0.559	0.000	0.368	0.364	−0.004	
5	1	0.000	0.453	0.549	0.000	0.452	0.548	0.294	0.295	0.001	0.004
	2	0.395	0.000	0.607	0.394	0.000	0.606	0.338	0.339	0.001	
	3	0.439	0.557	0.000	0.440	0.560	0.000	0.368	0.366	−0.002	
6	1	0.000	0.451	0.550	0.000	0.451	0.549	0.294	0.295	0.001	0.002
	2	0.393	0.000	0.608	0.393	0.000	0.607	0.338	0.339	0.001	
	3	0.439	0.559	0.000	0.440	0.560	0.000	0.368	0.367	−0.001	
7	1	0.000	0.450	0.550	0.000	0.450	0.550	0.294	0.294	0.000	0.001
	2	0.392	0.000	0.609	0.392	0.000	0.608	0.338	0.338	0.000	
	3	0.439	0.559	0.000	0.440	0.560	0.000	0.368	0.367	−0.001	
8	1	0.000	0.450	0.551	0.000	0.449	0.551	0.294	0.294	0.000	0.000
	2	0.391	0.000	0.609	0.391	0.000	0.609	0.338	0.338	0.000	
	3	0.440	0.560	0.000	0.440	0.560	0.000	0.368	0.368	0.000	

ing geosynchronous orbit, but can only view one location at a time. The time required to redirect the sensor is one time unit, and the site measurements are instantaneous once the sensor is in position. The goal is to find both IID and Markov sensor motion strategies that will best track the arrivals or departures of targets at each location. Site dynamics are given by:

$$A = \begin{bmatrix} 1.06 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.24 \end{bmatrix}.$$

To check to see if the coverage problem can be solved with the IID method, we use inequality (3) and find that, since the left hand side is not greater than five, the coverage problem cannot be solved with an IID sensor motion strategy.

$$\frac{1}{1.06^2} + \frac{1}{1.07^2} + \frac{1}{1.10^2} + \frac{1}{1.12^2} + \frac{1}{1.15^2} + \frac{1}{1.24^2} \approx 4.79 < 5$$

Using the Markov method, the equilibrium probabilities that minimize the objective function (15) are $\pi = [0.103\ 0.116\ 0.151\ 0.171\ 0.198\ 0.261]$. A transition probability matrix that leads to this π vector is given by

$$T = \begin{bmatrix} 0 & 0.113 & 0.154 & 0.181 & 0.220 & 0.332 \\ 0.100 & 0 & 0.157 & 0.184 & 0.223 & 0.336 \\ 0.105 & 0.121 & 0 & 0.192 & 0.233 & 0.349 \\ 0.109 & 0.124 & 0.169 & 0 & 0.240 & 0.358 \\ 0.114 & 0.130 & 0.177 & 0.207 & 0 & 0.372 \\ 0.131 & 0.149 & 0.202 & 0.235 & 0.283 & 0 \end{bmatrix},$$

which yields an objective function value of $0.995 < 1$, indicating that all sites are stable. In this example, the increased flexibility of the Markov sensor motion strategy is required; i.e., the sites cannot be successfully observed unless the sensor motion strategy at each time step accounts for the current position of the sensor.

SUMMARY AND FUTURE WORK

This paper presents methods for finding optimal sensor coverage given a model of the dynamics of an observed system. These methods might be used to give unmanned air vehicles the capability to make autonomous path planning decisions, or to help a mission planner determine the number of target areas that could be covered by a single vehicle. Results are presented for a single sensor using two sensor motion strategies. We demonstrated that both the IID model and the Markov chain model have unique advantages. The IID model requires fewer calculations to get a solution, but there are problems that cannot be solved with an IID sensor motion model, but can be solved with the increased flexibility of the Markov chain model. Conversely, if the dynamics of one site dominate to the extent that the sensor would need to spend more than one-half its time at that site, then the Markov Chain sensor motion model cannot be used, at least with the method presented here. The IID model has no such restriction. The results presented here build on feasibility criteria developed by Tiwari *et al.* (2005) and computational techniques developed by Yerrick *et al.* (2006).

All of our work to date is based on stochastic sensor motion strategies. These strategies have the inherent advantage of being less predictable to an observant enemy, with the disadvantage that there is some non-zero probability that a particular site will not be observed in any number of consecutive steps. This suggests that deterministic motion strategies may be desired in some cases. The development of such deterministic strategies is one area for future research.

Other areas for future research include formulations that consider multiple sensors or relax one or more of the underlying assumptions; e.g., statistically independent sites, uniform transition times, no cost of movement or measurement, or time-invariant models of system dynamics and measurement.

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APPENDIX

```
%*******
% Step 1: Let each row in T be the
  equilibrium probabilities
for i=1:numberOfSites
   for j=1:numberOfSites
      T(i,j) = piDesired(j);
      end
end
%*******
%Step 2: Each element on the diagonal of T is
  made to be zero, and
%all elements in each row are normalized so
  that each row sums to
%one
%Each diagonal element equal to zero
for i=1:numberOfSites
   T(i,i) = 0;
end
%Normalize elements in each row so that
  rows sum to one
Ttemp=T;
for i=1:Sites
   rowsum(i)=sum(T(i,:));
end
for i=1:numberOfSites
   for j=1:numberOfSites
      Ttemp(i,j) = T(i,j)/rowsum(i);
   end
end
%*******
%Step 3: Equilibrium probabilities of the
  current T are found
%using piCalculated = T*piDesired
piCalculated=piDesired*Ttemp;
%*******
%Step 4: Desired equilibrium probabilities are
  subtracted from
%calculated equilibrium probabilities :
  d=piCalculated − piDesired
d=piCalculated-piDesired;
%*******
dsum=sum(abs(d));
while dsum > .0000000000001
   %Step 5: New T is found : New T = Old T
   − d
   for i=1:numberOfSites
      for j=1:numberOfSites
      T(i,j) = Ttemp(i,j) − d(j);
      end
   end
%*******
%Step 6: Process is repeated until the sum of
  the absolute
%value of d is less than some predefined
  epsilon
%Make the diagonal elements zero
for i=1:numberOfSites
   T(i,i)=0;
end
%Normalize elements in each row so that
  rows sum to one
Ttemp=T;
for i=1:numberOfSites
   rowsum(i)=sum(T(i,:));
end
for i=1:numberOfSites
   for j=1:numberOfSites
      Ttemp(i,j) = T(i,j)/rowsum(i);
   end
end
piCalculated=piDesired*Ttemp;
d=piCalculated-piDesired;
dsum=sum(abs(d));
end
```

Optimal Placement of Suicide Bomber Detectors

Xiaofeng Nie

University at Buffalo (SUNY)
xnie2@buffalo.edu

Prof. Rajan Batta

University at Buffalo (SUNY)
batta@eng.buffalo.edu

Prof. Colin Drury

University at Buffalo (SUNY)
drury@buffalo.edu

Prof. Li Lin

University at Buffalo (SUNY)
lilin@eng.buffalo.edu

APPLICATION AREA:
Modeling, Simulation & Gaming and Strategic Operations

OR METHODOLOGIES:
Nonlinear Programming and Decision Analysis

ABSTRACT

This paper develops upon the work by Kaplan and Kress (2005), which considers the operational effectiveness of suicide bomber (SB) detector schemes. Here, we consider the optimal placement of detectors in a threat area where the potential targets are known. The threat area is divided into grids for the purpose of our analysis and can have several entrances. We assume that a SB would detonate at a potential explosive grid centroid. The number of individuals near every potential explosive grid is assumed to be given by a spatial Poisson process, with the density being a function of the specific potential explosive grid. It is assumed that the SB would take the shortest path from one of the entrances to the grid centroid where he/she intends to detonate. SB detectors are not perfectly reliable, with the probability of detection being a function of how long the SB would stay in the effective detection area. We choose the objective of minimizing the expected number of casualties. The problem is formulated as a nonlinear integer program and properties are derived to gain insights into the model as well as to develop efficient solution methods. Later, a greedy adding heuristic and a branch and bound algorithm are proposed. A base case is analyzed to illustrate the application of the model. We also perform a sensitivity analysis for a number of key factors as well as an investigation of the performance of the greedy heuristic procedure.

INTRODUCTION

Kaplan and Kress (2005) analyze the operational effectiveness of suicide bomber (SB) detector schemes. There, they consider two urban environments where SBs attack. The first environment is a grid model which presumes a layout of blocks separated by street and sidewalk. The second environment is a plaza model where the potential circular targets with radius τ are distributed in accordance with a spatial Poisson process. They discuss the expected number of casualties under two different possible interventions. The first intervention is to instruct the individuals to flee and the second one is to hit-the-deck. They conclude that under some situations, intervention may even increase the expected number of casualties.

We develop upon the work of Kaplan and Kress by considering SB detector placement in an environment where these detectors are not assumed to be fully reliable. We draw upon constructs/concepts from geometrical probability (c.f. Larson and Odoni (1981)) and from location science (c.f. Drezner and Hamacher (2002)) to do this. The framework we adopt is a combination of the grid and plaza models in Kaplan and Kress. We assume that there are several entrances and that the whole area is divided into grids, with some of them blocked, i.e., representing obstacles. We assume that we know the list of potential targets, that is, the set of grids where a SB would prefer to detonate. A grid in this set is called a potential explosive grid. The distribution of individuals near every potential explosive grid is assumed to be spatial Poisson, with the density being a function of the specific potential explosive grid. We assume that the SB would walk directly from an entrance to the chosen potential explosive grid. While doing so, he/she can not cross any blocked grids.

We assume that a SB detector is not perfectly reliable and that the probability of detection depends on how long the SB would stay in its effective detection area. The problem we consider is how to place SB detectors such that the expected number of casualties is minimized.

The paper is organized as follows. In the next section, we define the basic setting and develop the corresponding optimization model. The Properties Section derives several properties of the model. The Algorithms Section focuses on solution techniques, a greedy adding heuristic and a branch and bound algorithm. The Base Case Section contains a base case to illustrate the problem formulation. Computational analyses are presented in the Computational Analysis Section. Finally, our conclusions and future work directions are stated.

Basic Modeling

We assume that the threat area is rectangular and is divided into $m \times n$ grids, given by the set $G = \{j{:}j = 1, 2, \ldots, mn\}$ and

as shown in Figure 1. Entry into this threat area is possible through an entrance, where the set of entrances L is a subset of the set of boundary grids E. We let $l = |L|$ and label the entrances as $1, 2, \ldots, l$.

To model physical obstructions, we allow grids to be of two types. The first is a *blocked grid* through which the SB can not travel and in which no threatened individuals are present. The second is an *unblocked grid* through which SB travel is permitted and in which threatened individuals can also be present. Let $B \subset G$ be the set of all blocked grids, and $U = G \backslash B$ be the set of unblocked grids.

We assume knowledge of potential grids that the SB would like to attack. This set $S \subset U$ is referred to as the set of *potential explosive grids*. Near every potential explosive grid $j \in S$, threatened individuals are potentially present in accordance with a spatial Poisson process of density λ_j.

A SB uses entrance k and attacks potential explosive grid j with probability γ_{kj}. Hence, $\gamma_{kj} \geq 0$, $\forall k = 1, \ldots, l$, $j \in S$ and $\Sigma_{k=1}^{l} \Sigma_{j \in S} \gamma_{kj} = 1$. Our analysis assumes knowledge of these γ values. Later, we perform a robustness analysis across a range of γ values in the Computational Analysis Section. A fundamental assumption we make is that the SB can not detect a detector, i.e., the detectors are perfectly concealed.

The SB can walk directly from a grid center to another grid center provided that this straight line path does not intersect with a blocked grid. In other words, the SB attempts to travel on the straight line path from their chosen entrance k to their selected explosive grid j. If this straight line path intersects one or more blocked grids then they choose a shortest path, which consists of a sequence of straight line segments connecting grid centroids such that each such segment is nonintersecting with the set of blocked grids. To find such a path, we apply a shortest path algorithm (e.g. Dijkstra's Algorithm (Dijkstra, 1959)) from k to j using an inter-grid distance matrix whose entries are either (i) infinity if the straight line path between the two grid points intersects one or more blocked grids, or (ii) straight line distance.

We assume for the sake of simplicity in presentation that the shortest path from k to j is unique and is labeled as P_{kj}. The analysis is readily extendable to the situation of non-unique paths provided we make the assump-

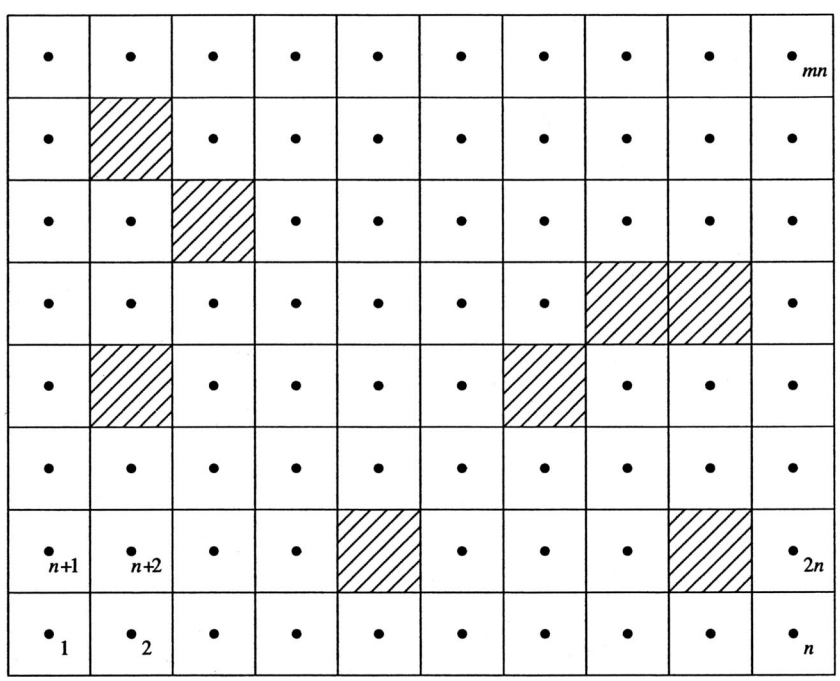

Figure 1. The area with $m \times n$ grids (The shadow grids are blocked).

tion that the SB randomly selects one of these paths.

We assume that grid centers for unblocked grids are potential detector placement points, and that the effective detection radius is τ, as determined by the National Research Council panel (NRC, 2004). As in Kaplan and Kress, we define timely detection as detection such that there are at least 10 seconds remaining before the SB reaches his/her targeted explosive grid. Assuming a walking speed of 1 m/sec, this converts to a distance of 10 m from the target.

Let $N_k(j)$ be the set of all non-blocked grids from which a detector can timely detect the SB while on path P_{kj}. To construct the set $N_k(j)$, we first identify all grids i such that the distance from i to the nearest point on path P_{kj} is less than τ. For each such i we then see if the first point of detection on path P_{kj} is at least 10 m away from j. If not, we remove i from the set. This first point of detection is easily obtained by constructing a circle of radius τ centered in grid i and observing its intersection with path P_{kj}. For simplicity in presentation, for the rest of the paper we will write SB detection when we mean timely SB detection.

Now, we give an example to show how to obtain the set $N_k(j)$ geometrically. In Figure 2, we assume that every grid is 10 m $\times$ 10 m and τ = 10 m. In this example, the entrance 1 is grid 3 and the explosive grid is grid 36. The shortest path from entrance 1 to grid 36 is 3 $\rightarrow$ 9 $\rightarrow$ 36. From the graph, if the center of the grid is within the detection zone which is defined as the set of all unblocked points from which a detector can timely detect the SB while on the shortest path, it belongs to the set $N_k(j)$. Thus, in this example, $N_1(36)$ = {3, 9, 15, 16, 22, 23, 29, 30, 35}.

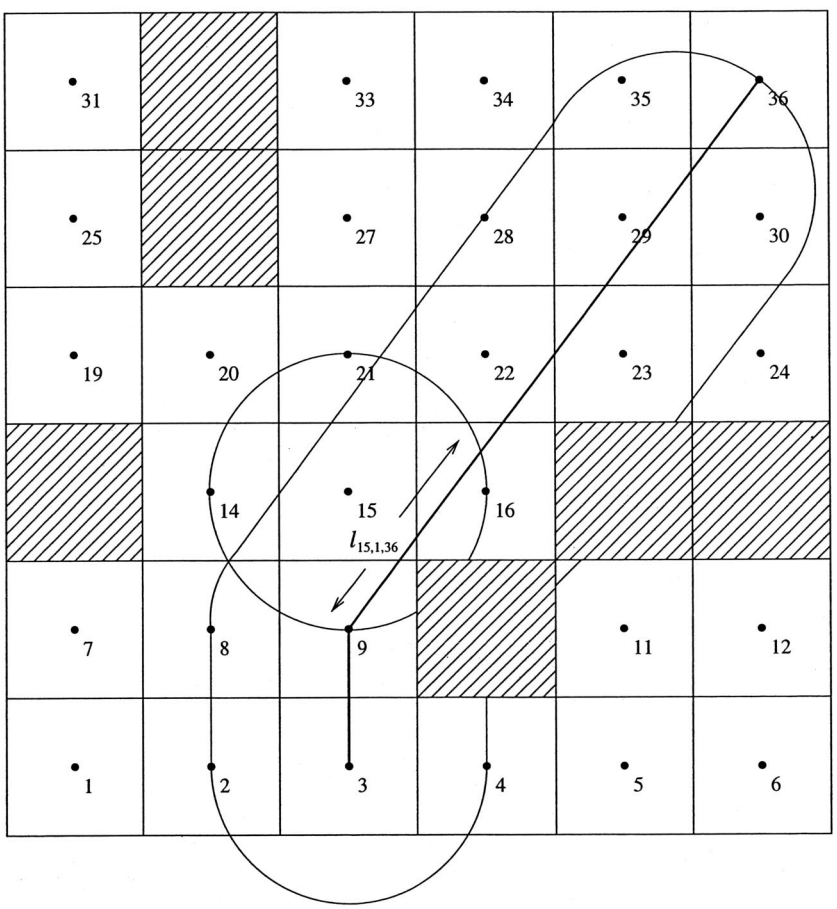

Figure 2. Illustration of $N_k(j)$.

We do not assume that the detectors are perfectly reliable. For every $i \in N_k(j)$, let p_{ikj} represent the probability of detecting the SB while traveling on path P_{kj}. To calculate p_{ikj}, we focus on the portion of path P_{kj} which is at least 10 m away from j. We label this sub-path as $\bar{P}_{kj}$. We now calculate the length of the section of $\bar{P}_{kj}$ which is within τ distance from i, l_{ikj}. These calculations are similar to those discussed in Batta and Chiu (1988) for routing of a vehicle carrying hazardous materials. If the entrance k lies in the circle centered at grid i with radius τ, we will extend the first segment of $\bar{P}_{kj}$ to intersect with the circle when calculating l_{ikj}.

As in Przemieniecki (2000), we assert that

$$p_{ikj} = 1 - e^{-\eta l_{ikj}}, \tag{1}$$

where η is the detector's instantaneous detection rate. For the example shown in Figure 2, given that $\eta = 0.06$, we have that $p_{15,1,36} = 1 - e^{-0.06\times 15.833} = 0.613$.

By the definition of $N_k(j)$, we can restrict our detector placements to belong to the set $T = \cup_{k,j} N_k(j)$. Since the determination of detector placements is our primary motive, we define, for each $j \in T$, a binary variable x_j as follows:

$$x_j = \begin{cases} 1 & \text{if there is a detector placed} \\ & \text{in the center of grid } j, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that there is at most one detector placed at a grid.

The expected number of casualties given that the SB detonates at grid j, C_j, is given by Equation 2 in Kaplan and Kress. This is needed for our objective function and is explained in the Base Case Section.

Consideration of detectors allows us to perform a suitable intervention, i.e., when a detector alarm goes off, an action can be taken. The specific action we consider is that of neutralizing the SB. We assume a success probability of θ for this action. Other interventions (e.g., instructing individuals to flee and hit-the-deck) are not considered in this paper because these are demonstrated to be ineffective in many cases by Kaplan and Kress. Figure 3 gives a breakdown of possible events.

We now focus on constructing the objective function. Our fundamental assumption is that the detectors work independently. The probability of non-detection along path P_{kj} is

$$Pr\{ND_{kj}\} = \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i},$$

in which case the resultant number of expected casualties is C_j. If detection occurs (with probability $1 - Pr\{ND_{kj}\}$), there is still a chance of non-neutralization resulting in $(1 - \theta)C_j$ expected number of casualties. Given the γ_{kj} values which specify the probabilities of using a specific entrance k and targeting a specific grid j, we get the total expected number of casualties as:

$$\sum_{k=1}^{l} \sum_{j \in S} [(1 - \theta)C_j\gamma_{kj} + \theta \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i} C_j \gamma_{kj}]. \tag{2}$$

We note that the first term of (2) is a constant and hence can be dropped from the perspective of optimization of detector placement. Furthermore, we let W_{kj} be equal to $\theta C_j \gamma_{kj}$.

For the optimization problem to be meaningful, we need to restrict the total number of detectors, M, to be placed. The problem can then be stated as the following nonlinear binary integer program:

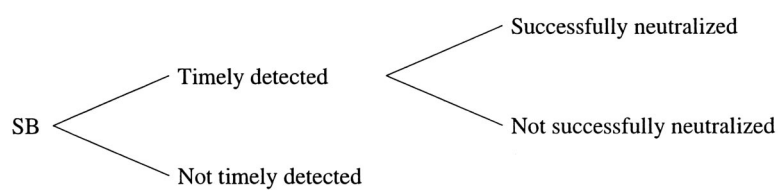

Figure 3. All possible events related to a SB.

$$(P) \quad \text{Min} \quad \sum_{k=1}^{l} \sum_{j \in S} W_{kj} \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i}$$
$$\text{s.t.} \quad \sum_{j \in T} x_j \leq M,$$
$$x_j \in \{0, 1\}, \forall j \in T.$$

PROPERTIES

In this section we develop a series of properties with the following goals in mind:

- Improved confidence level when using implicit enumeration;
- Better insight into the model and development of an alternative heuristic; and
- Dominance results that reduce the problem dimension/feasible set.

Implicit Enumeration Related Properties

The linear relaxation of (P) is:

$$(P_{LR}) \quad \text{Min} \quad \sum_{k=1}^{l} \sum_{j \in S} W_{kj} \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i}$$
$$\text{s.t.} \quad \sum_{j \in T} x_j \leq M,$$
$$0 \leq x_j \leq 1, \forall j \in T.$$

Clearly, (P_{LR}) is a convex program, which implies that we can obtain an optimal solution for (P_{LR}) using well known algorithms (e.g., sequential quadratic programming (SQP) approach (Boggs and Tolle, 1996)).

Model Insight and Heuristic Development

Here, we just state two simple properties without proof. Our first property establishes that all detectors will be deployed, when possible. The second property focuses on the situation where we have just one entrance and one potential explosive grid. The result is used in the development of a greedy heuristic in the Algorithms Section.

Property 1. If $|T| \geq M$, $\Sigma_{j \in T} x_j^* = M$ in the optimal solution X^*.

Property 2. Suppose there is only one entrance k and one potential explosive grid j. For the situation where $|N_k(j)| \leq M$, we have $x_i^* = 1$ for every $i \in N_k(j)$ in the optimal solution X^*. For the situation $|N_k(j)| > M$, an optimal solution X^* is found by setting to 1 the first M elements in the set $N_k(j)$ when it is arranged in decreasing order of p_{ikj}.

Dominance Results

We first provide a grid dominance definition which allows us to eliminate some potential detector locations.

Definition 1. Consider two grids u and v. If $p_{ukj} \geq p_{vkj}$ for all k, j pairs and $p_{ukj} > p_{vkj}$ for at least one k, j pair, we say that grid u *dominates* grid v.

Theorem 1. If grid v is dominated by grid u, $x_v^* \leq x_u^*$ in an optimal solution.

Proof. (By contradiction) Consider an optimal solution X^*, in which $x_v^* > x_u^*$. Since both x_v and x_u are binary variables, we have that $x_v^* = 1$ and $x_u^* = 0$. According to the definition of dominance, just by letting $x_u^{**} = 1$ and $x_v^{**} = 0$ and keeping values of other decision variables the same as X^*, we obtain another feasible solution X^{**} which has a smaller objective function value than that of X^*. That contradicts with the fact that X^* is an optimal solution.

Corollary 1. If grid v is dominated by at least M grids, $x_v^* = 0$ in an optimal solution.

Proof. Suppose that $x_v^* = 1$ in an optimal solution X^*. According to Theorem 1, $x_u^* = 1$ for all those grids u which dominate grid v. That solution does not satisfy the constraint $\Sigma_{j \in T} x_j \leq M$, which is a contradiction.

Corollary 2. If there exists one grid which dominates all other grids and $M \geq 1$, we will locate one detector in that grid in an optimal solution.

Proof. Suppose we do not locate one detector in that grid, according to Theorem 1, we have $\Sigma_{j \in T} x_j = 0$ in the optimal solution. That leads to a contradiction.

As a special case of Corollary 2, if there is only one entrance and several explosive grids and the entrance grid dominates all other grids,

we will place one detector in the entrance grid in the optimal solution, given that we will deploy at least one detector. This is intuitive.

ALGORITHMS

In this section, we propose a greedy adding heuristic and a branch and bound algorithm to solve (P).

Greedy Adding Heuristic

We propose a greedy adding heuristic to obtain an approximate solution to this optimization problem. The idea of this heuristic is as follows: we place the first detector in the grid such that it will minimize the expected casualties if we are allowed to deploy only one detector. Then we choose the second placement such that the expected casualties is minimized given that the position of the first detector is fixed. We continue to follow this rule until we place the Mth detector.

Greedy Adding Heuristic (GAH) Procedure:

- Step 1: The initial potential placement set is $P = T$ and initial placement set is $Q = \emptyset$.
- Step 2: For every $j \in P$, compute the total expected casualties if we just place an additional detector at grid j. Then compute

$$i \in argmin\{j \in P: \text{ total expected casualties if an additional detector is placed at grid } j\},$$

and let

$$P = P\backslash\{i\} \text{ and } Q = Q \cup \{i\}.$$

- Step 3: Check if $|Q| < M$, if yes, then go to step 2, otherwise, Q represents the selected detector placements.

We now investigate a special case where the GAH procedure yields an optimal solution. This is for the situation where none of the sets $N_k(j)$ intersect. More formally, the condition is that for all possible pairs of (k, j), $\cap N_k(j) = \emptyset$.

Let g be the number of distinct $N_k(j)$, labeled as 1, 2, . . ., g; let $a_m = W_{kj}$ for the mth $N_k(j)$; let h_m be the number of elements in the mth $N_k(j)$ set; and q_{mn} be the p_{ikj} value for the nth element in the mth $N_k(j)$ set. With this notation in place, we recognize that (P) is reduced to the following optimization problem:

$$(A) \quad \text{Min} \quad \sum_{m=1}^{g} a_m \prod_{n=1}^{h_m} (1 - q_{mn})^{x_{mn}}$$

$$\text{s.t.} \quad \sum_{m=1}^{g} \sum_{n=1}^{h_m} x_{mn} \le M,$$

$$x_{mn} \in \{0,1\}, \forall mn.$$

We therefore focus our attention on showing that the GAH procedure solves (A) optimally. To do this, we need to establish two properties. The first is a greedy choice property and the second is an optimal substructure property (Cormen et al., 2001).

Lemma 1. Greedy Choice Property: Let x_{rs} be the first decision variable to be set to be 1 by the GAH procedure. Then, there exists an optimal solution X^* to (A) where $x^*_{rs} = 1$.

Proof. (By construction) Let X^* be an optimal solution of problem (A). If x^*_{rs} happens to be 1, we are done. If $x^*_{rs} = 0$ in X^*, then we have two cases. In Case 1, there exists one rv such that $x^*_{rv} = 1$, where $v \in \{1, 2, \ldots, h_r\}\backslash\{s\}$. Case 2 considers the case where there does not exist such a rv such that $x^*_{rv} = 1$.

Since x_{rs} is the first decision variable set to be 1 by the GAH procedure, we have the following inequality

$$a_r(1 - q_{rs}) + \sum_{k \ne r} a_k \le a_c(1 - q_{cd}) + \sum_{k \ne c} a_k, \ \forall (c, d). \quad (3)$$

For Case 1, let us construct another feasible solution X^{**}. The only difference between X^{**} and X^* is that $x^{**}_{rs} = 1$ and $x^{**}_{rv} = 0$ in X^{**}. From Inequality (3), let $c = r$ and $d = v$, we have that $a_r(1 - q_{rs}) \le a_r\,(1 - q_{rv})$. Thus, we have

$$a_r \prod_{n \notin \{s,v\}} (1 - q_{rn})^{x^*_{rn}}(1 - q_{rs}) \le a_r \prod_{n \notin \{s,v\}} (1 - q_{rn})^{x^*_{rn}}(1 - q_{rv}),$$

which leads to the conclusion that the objective function value under X^{**} is no greater than that

of X^*. Hence, X^{**} is an optimal solution with $x^{**}_{rs} = 1$.

For Case 2, suppose that in X^*, we have $x^*_{ut} = 1$, where $u \neq r$ and $t \in \{1, 2, \ldots, h_u\}$. Let us construct another feasible solution X'. The only difference between X' and X^* is that $x'_{rs} = 1$ and $x'_{ut} = 0$ in X'. In order to prove that X' is also an optimal solution, what we need to prove is that

$$a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}} + a_r(1 - q_{rs}) \leq a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}}(1 - q_{ut}) + a_r,$$

that is,

$$a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}} q_{ut} \leq a_r q_{rs}. \tag{4}$$

From Inequality (3), let $c = u$ and $d = t$, we have that $a_r(1 - q_{rs}) + a_u \leq a_u(1 - q_{ut}) + a_r$, that is, $a_u q_{ut} \leq a_r q_{rs}$. Since $\Pi_{n \neq t} (1 - q_{un})^{x^*_{un}} \leq 1$, (4) is correct. Hence, in this case, X' is also an optimal solution with $x'_{rs} = 1$. The result follows.

Lemma 2. Optimal Substructure Property: If X^* is an optimal solution to problem (A) containing $x^*_{rs} = 1$, then the remaining elements of X^* (with x^*_{rs} deleted) are optimal to the remaining optimization problem (A'):

$$\begin{aligned} (A') \quad \text{Min} \quad & \sum_{m \neq r} a_m \prod_{n=1}^{h_m} (1 - q_{mn})^{x_{mn}} + a_r(1 - q_{rs}) \prod_{n \neq s} (1 - q_{rn})^{x_{rn}} \\ \text{s.t.} \quad & \sum_{mn \neq rs} x_{mn} \leq M - 1, \\ & x_{mn} \in \{0,1\}, \forall mn \neq rs. \end{aligned}$$

Proof. (By contradiction) Suppose the remaining X^* with x^*_{rs} deleted is not optimal to problem (A'). Then, there exists an optimal solution $\bar{X}$ to (A'). By combining this optimal solution $\bar{X}$ with $x_{rs} = 1$, we obtain another feasible solution to (A) with objective function value less than that of X^*, which is a contradiction.

Theorem 2. If for all possible pairs of (k,j), $\cap N_k(j) = \emptyset$, then the solution given by the GAH procedure is optimal.

Proof. By combining the results of Lemma 1 and Lemma 2, the theorem follows.

Branch and Bound Algorithm

Since the relaxation problem is a convex nonlinear program, we can use a branch and bound solution algorithm and obtain an exact solution (Gupta and Ravindran, 1985).

To enhance the performance of the solution method, we can use Corollary 1 of Theorem 1 to check if a grid is dominated by at least M other grids. If yes, we can eliminate the decision variable associated with that grid. Moreover, according to Theorem 1, we can add constraint $x_v \leq x_u$ to the constraint set if we know that grid v is dominated by grid u. By doing this, we decrease the feasible space without eliminating any optimal solutions.

BASE CASE

In this section, we will provide a base case to illustrate the problem formulation and to investigate the performance of the proposed GAH procedure which is measured by the relative error. Our case is based on a 80 m × 80 m study area, which is divided into 64 equal grids of size 10 m × 10 m (see Figure 4). In Table 1, we summarize the parameter values of our base case. The values of the parameters τ, r and b are chosen consistent with those in Kaplan and Kress.

In this table, the target-area radius, individual base width and number of effective fragments are needed to calculate the C_j values using Equation 2 in Kaplan and Kress which is restated as follows:

$$C_j = \frac{2\pi}{\lambda_j b^2} (1 - (1 + \lambda_j br)e^{-\lambda_j br}).$$

For this base case, we assume that the densities near two explosive grids are equal and that the probabilities γ_{kj} are equal for every entrance and explosive grid pair. From the parameter values, we can obtain W_{kj} for each (k, j) pair.

In the base case, the shortest paths from each entrance to each potential explosive grid are shown in Figure 4, which can be obtained via some standard shortest path algorithm (e.g. Dijkstra's Algorithm). For example, the shortest

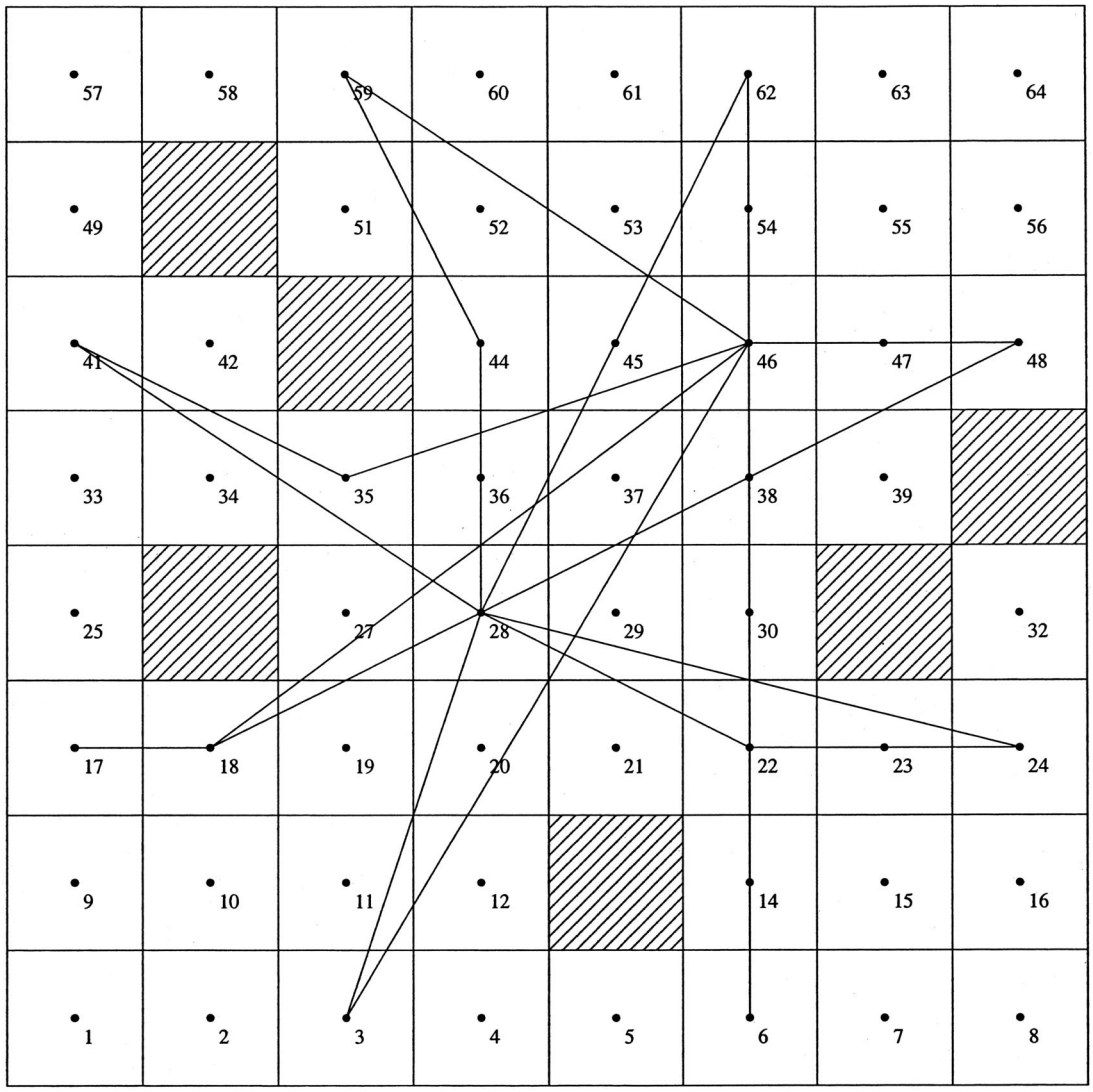

Figure 4. The base case with 8 × 8 grids.

path from entrance grid 41 to grid 46 is 41 → 35 → 46.

After we obtain the shortest paths, we can obtain the corresponding $N_k(j)$ for each entrance and potential explosive grid combination. For example, here

$$N_1(46) = \{2, 3, 4, 11, 12, 20, 21, 28, 29, 37, 38, 45\}.$$

For each $i \in N_k(j)$, we can obtain the corresponding l_{ikj}. Using Equation (1), we can obtain all these p_{ikj} values. For example, $l_{21,1,46} = 14.552$. Thus, $p_{21,1,46} = 1 - e^{-\eta \times l_{21,1,46}} = 0.582$.

By using the GAH procedure, we will first choose grid 30, then grid 37 and grid 59. The corresponding objective function value is 27.98.

When we use the branch and bound algorithm to solve this base case, there are $|T| = |\cup_{k,j} N_k(j)| = 47$ decision variables if we do not use dominance properties. By using the Corollary 1 of Theorem 1, we can eliminate some number of decision variables. For example, grids 2, 4 are dominated by grids 3, 11, 12, grids 16, 32 are dominated by grids 22, 23, 24 and grids 61, 63

Table 1. Base case parameter values

Parameter	Description	Value
B	Set of blocked grids	$B = \{13, 26, 31, 40, 43, 50\}$
L	Set of entrances (labeled as 1, 2, . . . , 8)	$L = \{3, 6, 17, 24, 41, 48, 59, 62\}$
S	Set of potential explosive grids	$S = \{28, 46\}$
M	Number of detectors to be placed	3
γ_{kj}	Probability that a SB enters from k and attacks j	0.0625
τ	Detector detection radius	10 m
η	Instantaneous detection rate	0.06
λ_j	Population density near grid j	$\lambda_{28} = \lambda_{46} = 0.4$ persons m^{-2}
θ	Probability of successful neutralization	0.6
r	Target-area radius	10 m
b	Individual base width	0.5 m
n	Number of effective fragments	∞
C_j	Expected casualties if explosion is at j	37.32

are dominated by grids 53, 54, 62. Therefore, we can eliminate decision variables x_2, x_4, x_{16}, x_{32}, x_{61}, x_{63}. Furthermore, we can use Theorem 1 to add some constraints to decrease the feasible space. For example, 51, 52, 58, 60 are dominated by grid 59. Thus we can add the following constraints:

$$x_{51} \leq x_{59},\ x_{52} \leq x_{59},\ x_{58} \leq x_{59},\ x_{60} \leq x_{59}.$$

When solved by a branch and bound algorithm, the optimal solution is $x^*_{22} = x^*_{37} = x^*_{59} = 1$ and the corresponding objective function value is 27.86. For this example, the relative error of the GAH procedure is

$$RE = \frac{\text{GAH Value} - \text{Branch and Bound Value}}{\text{Branch and Bound Value}} = \frac{27.98 - 27.86}{27.86} = 0.43\%.$$

COMPUTATIONAL ANALYSIS

In this section, we will first perform sensitivity and robustness analyses on the base case. Later, we perform an experiment to illustrate the performance of the GAH procedure.

Sensitivity and Robustness Analyses

Here, we study the sensitivity due to base case parameter value settings. First, we investigate the effect of the number of detectors. As shown in Figure 5, when we employ more detectors, the expected casualties will decrease. Moreover, the marginal benefit associated with each additional detector is decreasing, which makes sense intuitively.

Figure 6 illustrates the effect of detection radius. The line shown in the figure is a piecewise convex function, where a detection radius 10 m represents a break point. When the detection radius increases from 6 m, the marginal benefit is decreasing. But when the detection radius hits 10 m, the marginal benefit increases and later decreases.

Figure 7 illustrates the effect of the instantaneous detection rate. The trend makes sense, since when the detection rate increases the detector is more accurate, hence the expected casualties will decrease. Figure 8 illustrates the effect of the target radius. When the target radius increases, the marginal destruction will actually decrease.

We now consider a sample robustness analysis for the base case. In the base scenario, we assume that all the γ_{kj} values are the same and are equal to 0.0625. The optimal solution is: $x^*_{22} = x^*_{37} = x^*_{59} = 1$. Here, we do one perturbation on γ_{kj}. We assume that $\gamma_{kj} = 0.055 + \epsilon_{kj}$, where ϵ_{kj} is a random term. The summation of ϵ_{kj} over all (k, j) pairs equals to $1 - 0.055 \times 16 = 0.12$. For the perturbation, we consider ten randomly generated situations. The corresponding γ_{kj} values and the optimal solution for each situation are summarized in Table 2. In this

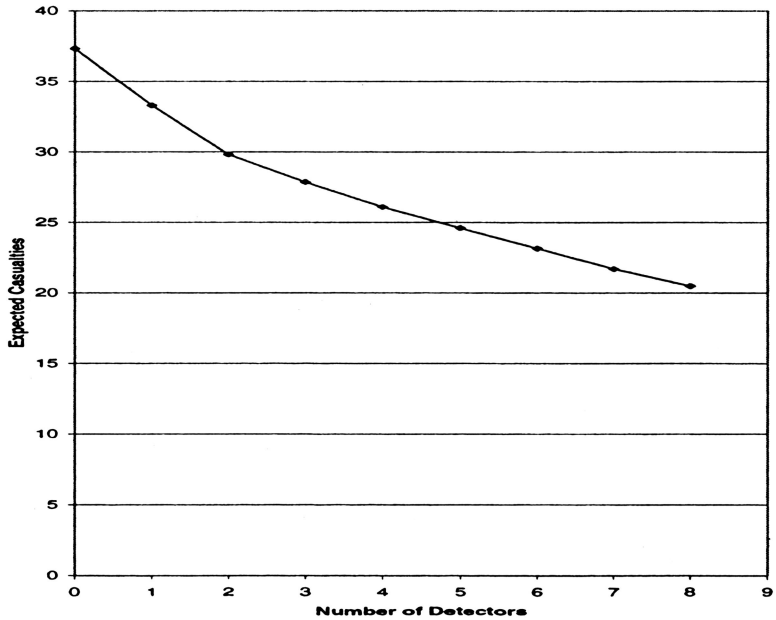

Figure 5. Effect of number of detectors.

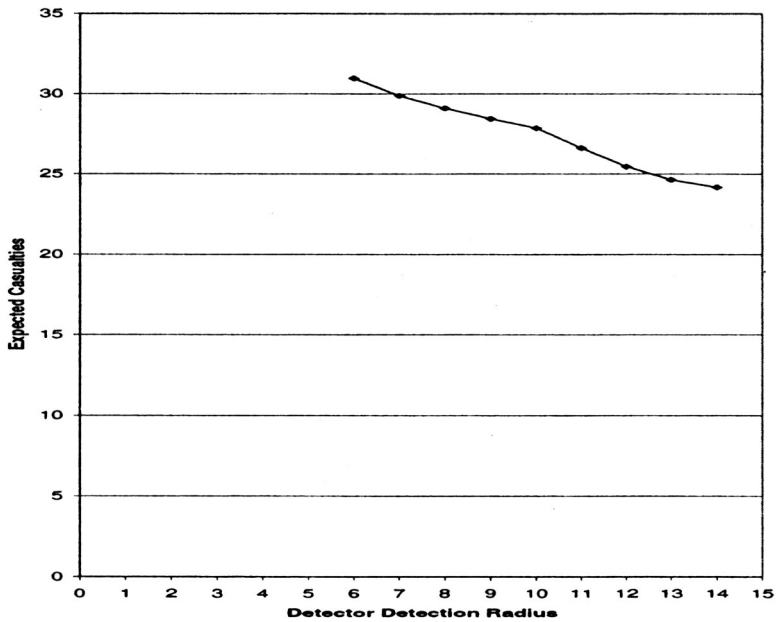

Figure 6. Effect of detection radius.

table, Situ represents situation. From this table, we can see that 9 out of 10 situations have grids 22, 37, 59 as optimal placements. Thus we can conclude that this placement set is a good choice when deploying three detectors.

To further explore the robustness of the sensor placements suggested in the base case, we consider two other cases where the values of γ_{kj} are randomly generated. For both these cases, we perform a perturbation analysis and assume that the summation of the random terms is still 0.12. The results show that in all 10 situations the same optimal placements for sensors are suggested.

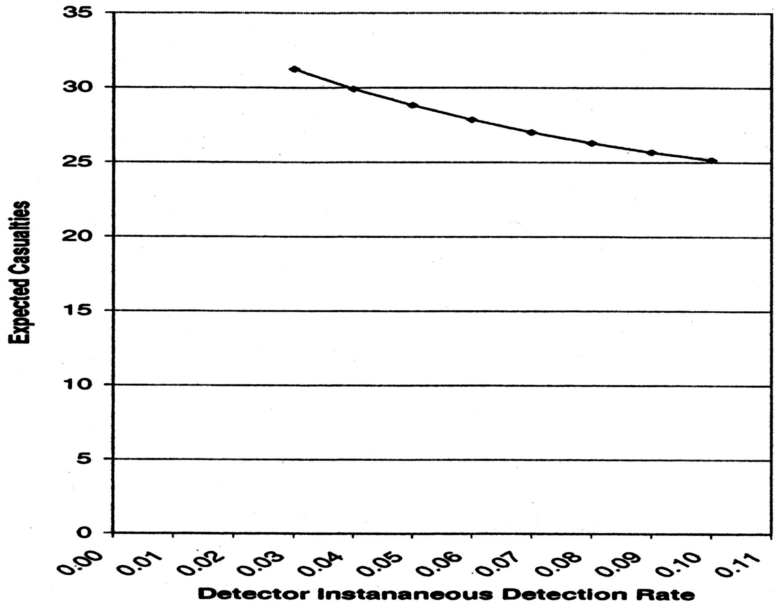

Figure 7. Effect of instantaneous detection rate.

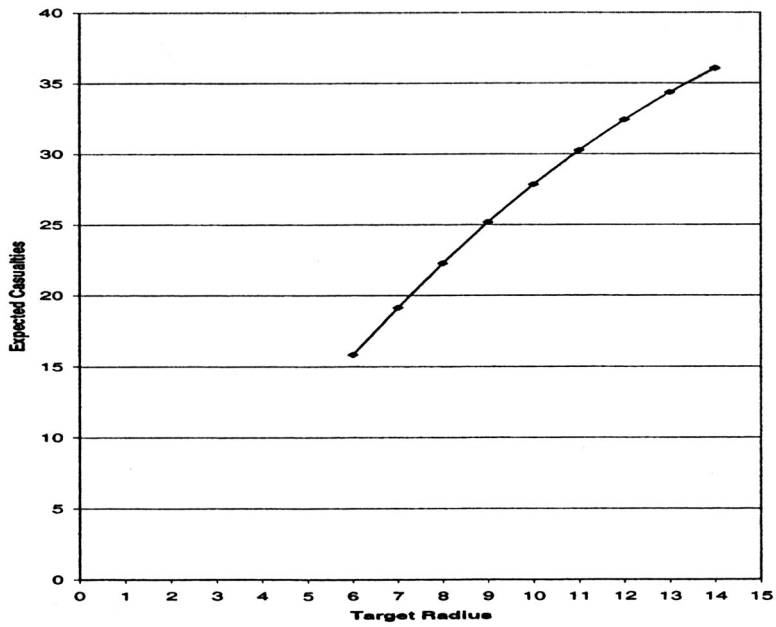

Figure 8. Effect of target radius.

Performance of the GAH Procedure

In this subsection, we conduct a computational experiment to illustrate the performance of the GAH procedure, while considering three factors. These are: (i) the number of entrances; (ii) the number of potential explosive grids; and (iii) whether or not we have blocked grids. For the number of entrances and the number of explosive grids, we consider 3 cases each. So, in all we have $3 \times 3 \times 2 = 18$ combinations.

Table 2. Robustness analysis with $\gamma_{kj} = 0.055 + \epsilon_{kj}$

γ_{kj}	Situ1	Situ2	Situ3	Situ4	Situ5	Situ6	Situ7	Situ8	Situ9	Situ10
$\gamma_{1,28}$	0.063	0.067	0.071	0.067	0.058	0.063	0.058	0.058	0.060	0.066
$\gamma_{1,46}$	0.068	0.063	0.066	0.060	0.058	0.055	0.055	0.062	0.059	0.058
$\gamma_{2,28}$	0.061	0.067	0.069	0.066	0.060	0.055	0.069	0.069	0.064	0.063
$\gamma_{2,46}$	0.058	0.060	0.071	0.066	0.062	0.061	0.065	0.066	0.066	0.059
$\gamma_{3,28}$	0.068	0.064	0.063	0.063	0.063	0.064	0.063	0.061	0.064	0.060
$\gamma_{3,46}$	0.061	0.064	0.062	0.058	0.061	0.058	0.058	0.057	0.060	0.061
$\gamma_{4,28}$	0.068	0.064	0.056	0.065	0.059	0.072	0.060	0.056	0.067	0.058
$\gamma_{4,46}$	0.066	0.056	0.062	0.063	0.064	0.057	0.061	0.063	0.060	0.059
$\gamma_{5,28}$	0.062	0.066	0.060	0.063	0.063	0.070	0.064	0.065	0.063	0.066
$\gamma_{5,46}$	0.062	0.068	0.055	0.061	0.063	0.055	0.062	0.062	0.058	0.066
$\gamma_{6,28}$	0.059	0.057	0.057	0.067	0.065	0.064	0.063	0.067	0.067	0.065
$\gamma_{6,46}$	0.056	0.068	0.061	0.059	0.064	0.060	0.068	0.056	0.064	0.068
$\gamma_{7,28}$	0.056	0.062	0.068	0.065	0.066	0.073	0.061	0.061	0.057	0.061
$\gamma_{7,46}$	0.067	0.056	0.061	0.056	0.065	0.058	0.066	0.069	0.064	0.068
$\gamma_{8,28}$	0.068	0.062	0.061	0.059	0.065	0.072	0.057	0.063	0.059	0.059
$\gamma_{8,46}$	0.057	0.057	0.057	0.062	0.065	0.062	0.070	0.067	0.066	0.062
Optimal Sol	22,37,59	22,36,37	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59

The basic setting is shown in Table 3. In this table, NE represents the number of entrances, NEx represents the number of explosive grids, EGs represents the set of explosive grids, W/Wo represents with or without blocked grids, BGs represents the set of blocked grids, GSol represents the solution of GAH procedure, GVal represents the objective function value from the GAH procedure, GT represents the running time (in CPU seconds) of the GAH procedure, BBSol represents the solution from the branch and bound algorithm, BBVal represents the objective function value from the branch and bound algorithm, BBT represents

Table 3. Details of computational experiment

NE	Entrances	NEx	EGs	W/Wo	BGs	GSol	GVal	GT	BBSol	BBVal	BBT	RE
8	3,6,17,24,41,48,59,62	3	22,28,46	W	13,26,31,40,43,50	20,30,36	28.14	0.13	20,30,36	28.14	164	**0.00%**
8	3,6,17,24,41,48,59,62	3	22,28,46	Wo	N/A	21,30,36	30.20	0.08	21,30,36	30.20	1146	**0.00%**
8	3,6,17,24,41,48,59,62	2	28,46	W	13,26,31,40,43,50	30,37,59	27.98	0.07	22,37,59	27.86	172	0.43%
8	3,6,17,24,41,48,59,62	2	28,46	Wo	N/A	30,37,44	30.00	0.08	30,37,44	30.00	646	**0.00%**
8	3,6,17,24,41,48,59,62	1	28	W	13,26,31,40,43,50	19,30,44	27.29	0.04	19,30,44	27.29	28	**0.00%**
8	3,6,17,24,41,48,59,62	1	28	Wo	N/A	19,30,44	28.76	0.07	19,30,44	28.76	92	**0.00%**
6	3,6,24,33,59,62	3	22,28,46	W	13,26,31,40,43,50	3,30,36	27.83	0.02	20,30,44	27.74	234	0.32%
6	3,6,24,33,59,62	3	22,28,46	Wo	N/A	21,44,62	29.21	0.05	3,6,44	29.09	435	0.42%
6	3,6,24,33,59,62	2	28,46	W	13,26,31,40,43,50	3,22,44	26.30	0.02	3,22,44	26.30	163	**0.00%**
6	3,6,24,33,59,62	2	28,46	Wo	N/A	21,44,62	29.12	0.06	20,24,44	28.80	743	1.13%
6	3,6,24,33,59,62	1	28	W	13,26,31,40,43,50	3,22,44	25.52	0.03	3,22,44	25.52	26	**0.00%**
6	3,6,24,33,59,62	1	28	Wo	N/A	3,21,44	27.11	0.03	3,21,44	27.11	82	**0.00%**
4	4,24,33,61	3	22,28,46	W	13,26,31,40,43,50	4,30,33	25.93	0.03	24,34,61	25.59	97	1.35%
4	4,24,33,61	3	22,28,46	Wo	N/A	4,32,33	25.59	0.07	4,33,61	25.59	79	**0.00%**
4	4,24,33,61	2	28,46	W	13,26,31,40,43,50	30,33,61	24.56	0.01	30,34,61	24.56	90	**0.00%**
4	4,24,33,61	2	28,46	Wo	N/A	4,33,61	25.59	0.07	4,33,61	25.59	101	**0.00%**
4	4,24,33,61	1	28	W	13,26,31,40,43,50	4,32,61	25.59	0.02	12,34,61	25.59	195	**0.00%**
4	4,24,33,61	1	28	Wo	N/A	4,32,61	25.59	0.01	4,24,61	25.59	126	**0.00%**

the running time (in CPU seconds) of the Branch and Bound algorithm, and RE represents the relative error of the GAH procedure.

From this experiment, we see that for 13 out of 18 combinations, the GAH procedure obtains the optimal solution. For the other five combinations, the relative error is very small, never more than 1.5%. Furthermore, the running time of the heuristic is 3 or more orders of magnitude smaller than that of the exact branch and bound algorithm. We note that this significantly reduced running time is especially helpful to conduct robustness analysis of the type discussed earlier in this section.

We now consider two situations where the grid sizes are 5 m $\times$ 5 m and 2.5 m $\times$ 2.5 m, respectively. For the first situation which has 256 grids, the GAH procedure only needs 0.52 seconds to obtain the solution. Even for the second case with 1024 grids, the running time for the GAH procedure is only 19.96 seconds. The branch and bound algorithm is not able to find an optimal solution for either of these situations even after several hours of effort. When grid size gets smaller, the model is a closer representation of reality but the dimension of the problem increases dramatically. For such cases, we can use the GAH procedure to obtain a good approximate solution.

CONCLUSIONS AND FUTURE WORK

In this paper, we considered how to deploy SB detectors in a threat area where the potential targets are known. Based on a grid model, we proposed an optimization model where the objective function is the total expected casualties. We derived a series of properties to gain a better understanding of the model. Later, we developed two algorithms (one heuristic, one exact) to solve the corresponding model. We also presented a base case study. Using this base case, we illustrated both sensitivity and robustness analyses of the model. Computational experiments to verify the effectiveness of the heuristic procedure were also developed.

In our model development we considered just one type of detector. Actually there are several kinds of detectors available in the market with different characteristics and costs. One possible future direction is to consider how to choose from different kinds of detectors, how many for each kind to employ, and where to site the different detector types.

In this paper, we assumed that the detectors work independently. This assumption is reasonable if the effective detection areas of each placed detector do not substantially intersect. For the case where these areas have significant intersection, the joint detection probability should be considered. In particular we note that the case of multiple sensors at a grid would fall in this category. The use of data fusion techniques to study the benefit of fused reports from multiple detectors is suggested as a way to address this modeling enhancement.

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About our Authors

A HOLISTIC APPROACH TO U.S. ARMY RECRUITING INPUT ALLOCATION

by John R. Brence, Michael J. Kwinn, Jr., and David A. Thomas

John Brence is the Chief of Research, Analysis and Technical Support for the U.S. Army CERT. His team is the Army's premiere technical research and analysis center focused on computer network defense (CND). In a former life, he was an Assistant Professor of Systems Engineering at the United States Military Academy at West Point. He loves to golf, but golf does not love him.

Mike directs the Systems Engineering Program in the Department of Systems Engineering at the United States Military Academy. He conducts research on military personnel management, total force and system development and combat assessment. He is a member of the Board of Directors of MORS and President, The Friends4Michael Foundation. A generally happy-go-lucky individual, his moods can usually be predicted by the fortunes of the Army and Miami Dolphin football teams.

Dave leads all the client-facing departments at Loyaltyworks, Inc.: Strategic Services, offering database marketing, analytics, measurement and research solutions; Account Management, providing thought leadership and execution for over 130 clients; Creative Services, developing vibrant marketing communications and creative for both internal and external audiences, Technology, providing technology solutions that provide the platform for our service delivery, and Delivery Management, continually improving our process for delivery excellence. Dave searches for balance under sail whenever he gets the chance.

BAYESIAN STOCKPILE RELIABILITY METHODOLOGY FOR COMPLEX SYSTEMS

by Christine M. Anderson-Cook, Todd Graves, Michael Hamada, Nicholas Hengartner, Valen E. Johnson, C. Shane Reese, and Alyson G. Wilson

Christine Anderson-Cook is a technical staff member in the Statistical Sciences group at Los Alamos National Laboratory. She is a Fellow of the American Statistical Association and a Senior Member of the American Society for Quality. Her research interests include reliability, response surface methodology and design of experiments.

Todd Graves is a technical staff member in the Statistical Sciences group at Los Alamos National Laboratory. He is a member of the American Statistical Association. He is the creator of the software system for statistical analysis using Markov chain Monte Carlo (MCMC).

Michael Hamada is a technical staff member in the Statistical Sciences group at Los Alamos National Laboratory. He is a Fellow of the American Statistical Association. His research interests include design of experiments, reliability and quality control.

Nicholas Hengartner is the Acting Group Leader of the Discrete Simulations Sciences group at Los Alamos National Laboratory. He is a member of the American Statistical Association. His research interests include reliability as well as statistical and probabilistic modeling.

Valen Johnson is a professor of biostatistics at the University of Texas M.D. Anderson Cancer Center. He is a fellow of the American Statistical Association and the Royal Statistical Society. His research interests include Bayesian model testing, Bayesian model assessment and diagnostics, ordinal data modeling, and models for dependent proportions and reliability modeling.

C. Shane Reese is an Associate Professor of Statistics at Brigham Young University. He is NOT a fellow of the American Statistical Association, the Royal Statistical Society, or the American Society for Quality, but his kids think he's pretty cool. His research includes Bayesian methods for reliability, computer experiments, sports statistics, and adaptive design algorithms.

Alyson Wilson is a technical staff member and the technical lead for DoD programs in the Statistical Sciences Group at Los Alamos National Laboratory. She is the chair of the American Statistical Association's (ASA) Section on Statistics in Defense and National Security and the chair of the ASA President's Task Force on Defense and Security. Her research interests include reliability, information integration, and Bayesian methods.

REDUCING AIRCRAFT DOWN FOR LACK OF PARTS WITH SPORADIC DEMAND

by Tovey C. Bachman

Tovey Bachman has been a Research Fellow at LMI since 1989. Originally trained as a mathematician at the University of Michigan, his current interests include readiness-based sparing, supply chain management, space logistics, threat-based munitions requirements, decision analysis, stochastic processes, discrete optimization, simulation, and software engineering. He is a co-developer of a model used by NASA to estimate spare parts requirements for missions to the moon, Mars, and beyond. He teaches a course in government sector supply chain management, and has performed a wide variety of logistics and economic analyses, involving the DLA, U.S. Air Force, U.S. Army, U.S. Navy, NASA, FAA, Amtrak, OSD, and the U.S. Congress. In his spare time he enjoys fixing bicycles and changing the oil on his aging Subaru.

TWO SENSOR MOTION MODELS FOR THE DYNAMIC SENSOR COVERAGE PROBLEM

by Nathan P. Yerrick, Mesut Yavuz and David E. Jeffcoat

1Lt Nathan Yerrick is a flight test engineer with the 59th Test and Evaluation Squadron at Nellis Air Force Base, Nevada. In his previous assignment, he worked for the Air Force Research Lab at Eglin Air Force Base, Florida, where he completed a master's degree in Industrial and Systems Engineering from the University of Florida.

Dr. Mesut Yavuz is a visiting assistant professor of industrial and systems engineering at the University of Florida Research and Engineering Education Facility in Shalimar, Florida. He conducts research in applied optimization in close collaboration with personnel from the Air Force Research Lab at Eglin Air Force Base, Florida.

Dr. David Jeffcoat is a senior research engineer with the Air Force Research Lab. He has more than 30 years experience in industry and government in a variety of technical jobs. When he's not analyzing dynamic systems for the Air Force, he enjoys spending time with his very dynamic three-year-old grandson.

OPTIMAL PLACEMENT OF SUICIDE BOMBER DETECTORS

by Xiaofeng Nie, Rajan Batta, Colin Drury, and Li Lin

Xiaofeng Nie is a Ph.D. candidate in the Department of Industrial and Systems Engineering at University at Buffalo, SUNY. He obtained his M.Phil. in Systems Engineering and Engineering Management from the Chinese University of Hong Kong in 2002 and a B.Eng. in Civil Engineering from Harbin Institute of Technology in 2000. His research interests are in applied operations research and stochastic modeling.

Rajan Batta is Professor of Industrial and Systems Engineering at University at Buffalo, SUNY, and Associate Dean for Graduate Studies in the School of Engineering & Applied Sciences. His field of research is Operations Research, with an emphasis on the use of stochastic modeling and optimization tools in the areas of disaster mitigation and modeling, homeland security applications, and military applications. He currently serves as a Project Director (Colin Drury and Li Lin are co-investigators) for an NSF funded grant on airport security system inspection strategies.

Colin G. Drury is UB Distinguished Professor of Industrial and Systems Engineering at University at Buffalo, SUNY. For the past 15 years, he has worked extensively for the FAA in reducing human errors in aircraft inspection and maintenance, receiving national awards from the FAA and the Human Factors and Ergonomics Society for this work. For the aviation maintenance industry he developed a series of widely-used Best Practices Guides for Non-Destructive Inspection equipment. He is also co-investigator (with Rajan Batta and Li Lin) in

a NSF funded grant on airport security system inspection strategies.

Li Lin is Professor of Industrial and Systems Engineering at University at Buffalo, SUNY. His research areas of specialty include manufacturing and health care systems simulation, and concurrent engineering. Dr. Lin's research has been supported by the National Science Foundation (NSF), the U.S. Environmental Protection Agency (EPA), the Agency for Healthcare Research and Quality (AHRQ) and more than twenty industrial companies, including several hospitals. He is also co-investigator (with Rajan Batta and Colin Drury) in a NSF funded grant on airport security system inspection strategies.

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TABLE 1: APPLICATION AREAS & OR METHODS

Composite Group	APPLICATION AREA	OR METHODOLOGY
I. STRATEGIC & DEFENSE	Strategic Operations Nuclear Biological Chemical Defense Arms Control & Proliferation Air & Missile Defense	**Deterministic Operations Research** Dynamic Programming Inventory Linear Programming Multiobjective Optimization Network Methods Nonlinear Programming
II. SPACE/C41SR	Operational Contribution of Space Systems Battle Management/Command and Control ISR and Intelligence Analysis Information Operations/Information Warfare Countermeasures Military Environmental Factors	**Probabilistic Operations Research** Decision Analysis Markov Processes Reliability Simulation Stochastic Processes Queuing Theory
III. JOINT WARFARE	Unmanned Systems Land & Expeditionary Warfare Littoral Warfare/Regional Sea Control Strike Warfare Air Combat Analysis & Combat ID Special Operations and Irregular Warfare Joint Campaign Analysis	
IV. RESOURCES	Mobility & Transport of Forces Logistics, Reliability, & Maintainability Manpower & Personnel	**Applied Statistics** Categorical Data Analysis Forecasting/Time Series Multivariate Analysis Neural Networks Nonparametric Statistics Pattern Recognition Response Surface Methodology
V. READINESS & TRAINING	Readiness Analytical Support to Training Casualty Estimation and Force Health Protection	
VI. ACQUISITION	Measures of Merit Test & Evaluation Analysis of Alternatives Cost Analysis Decision Analysis	
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