# Warranty Analysis of Repairable Systems

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# Repairable Systems

- Main focus is repairable systems
  > A system is repairable if by replacing or repairing system components, the system can be returned to service.

  > After the repair, the system may be

    > As good as new (renewal process)

    > Worse than new ☹

    > Better than new ☺

  > Behavior of system after repair depends on repair actions and effects of replaced components.

  > A key characteristic of a repairable system is that an individual system may fail multiple times for the same or different causes.

# Warranty and Reliability Analysis

- Warranty analysis and reliability analysis of field data are closely related areas.
- The authors have used graphical Time Dependent Reliability (*TDR*) methods extensively for analyzing product field reliability data.
  - *TDR* is for use with repairable systems.
- *TDR* methods are also very useful for the analysis of warranty data.
  - Can isolate specific failure modes and associated cost data easily

# Time Dependent Reliability

- Graphical technique based on studying the cumulative occurrence of events of interest over time on individual systems
  - The overall behavior is represented by the average of events across systems at fixed times – the mean cumulative function (*MCF*).
  - Can change definition of events of interest to suit study
    - Can be all field failures for field reliability
    - Can be warranty claims for warranty data analysis
    - Can be costs of repairs or claims
  - *TDR* approach is consistent with warranty analysis techniques as suggested by Kalbfleisch, Lawless and Robinson (1991) and also Nelson (2004).

# Estimating the *MCF*

- Consider a collection of *M* observed times of events $t_k$ for a set of units sold since a start time $t_0$
  - > Assume the event times are set in increasing order so $t_0 \leq t_1 \leq t_2 \leq \ldots \leq t_M$
    - > Note times may be calendar dates or may represent time in operation, e.g., system age
  - > The number of units active at any time is variable due to manufacture and sale at different times.
    - > Let $N(t_k)$ be the number of units active at the time represented by $t_k$

# Estimating the *MCF* - 2

- The Mean Cumulative Function (*MCF*) is defined successively as

$$MCF(t_0) = 0$$

$$MCF(t_k) = MCF(t_{k-1}) + \frac{w(t_k)}{N(t_k)}$$

where $w(t_k)$ is an incremental weight

- Incremental weight $w(t_k)$ may be
  - Number of failures at time $t_k$
  - Costs of failures at time $t_k$

# Extending the Analysis

- The *TDR* technique can be used to graphically explore the modes of failure by re-defining the events of interest.
  - For example, we can produce a MCF for each failure mode or failing component.
  - The resulting chart is a ***dynamic Pareto chart*** of failing components.
    - Each of these curves should fall beneath the overall MCF curve.
    - These component curves should add up to produce the overall MCF curve.
    - Again, cost data can be incorporated by weighting the individual component fails by their repair cost – scaled in dollars.

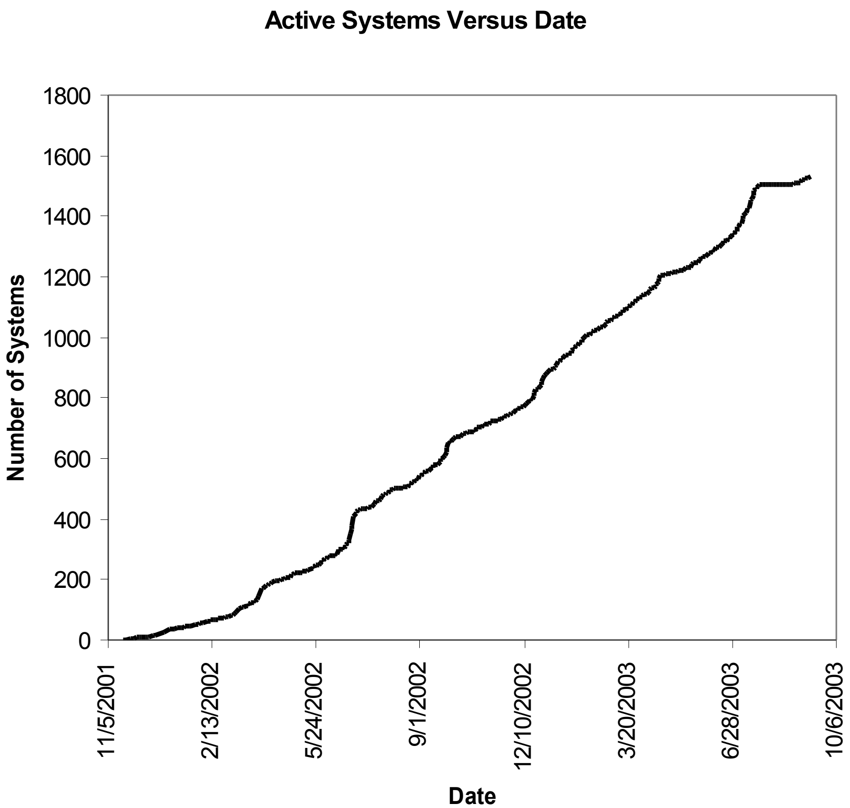
# Example

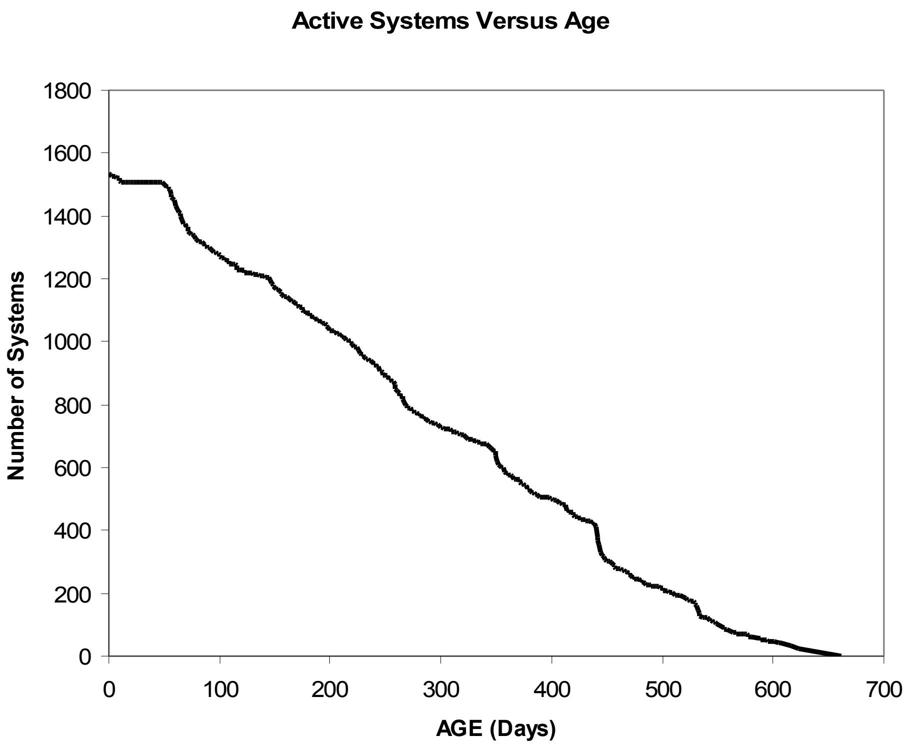
- This example contains roughly 1500 systems manufactured and installed over approximately a two year period.
- The data represent all systems manufactured over this period.
- Each system consists of many components, but four of these components are responsible for a large fraction of the failures.
- Adjustments were made for missing data. (See Glosup, 2002)

# Age Versus Date Dependency

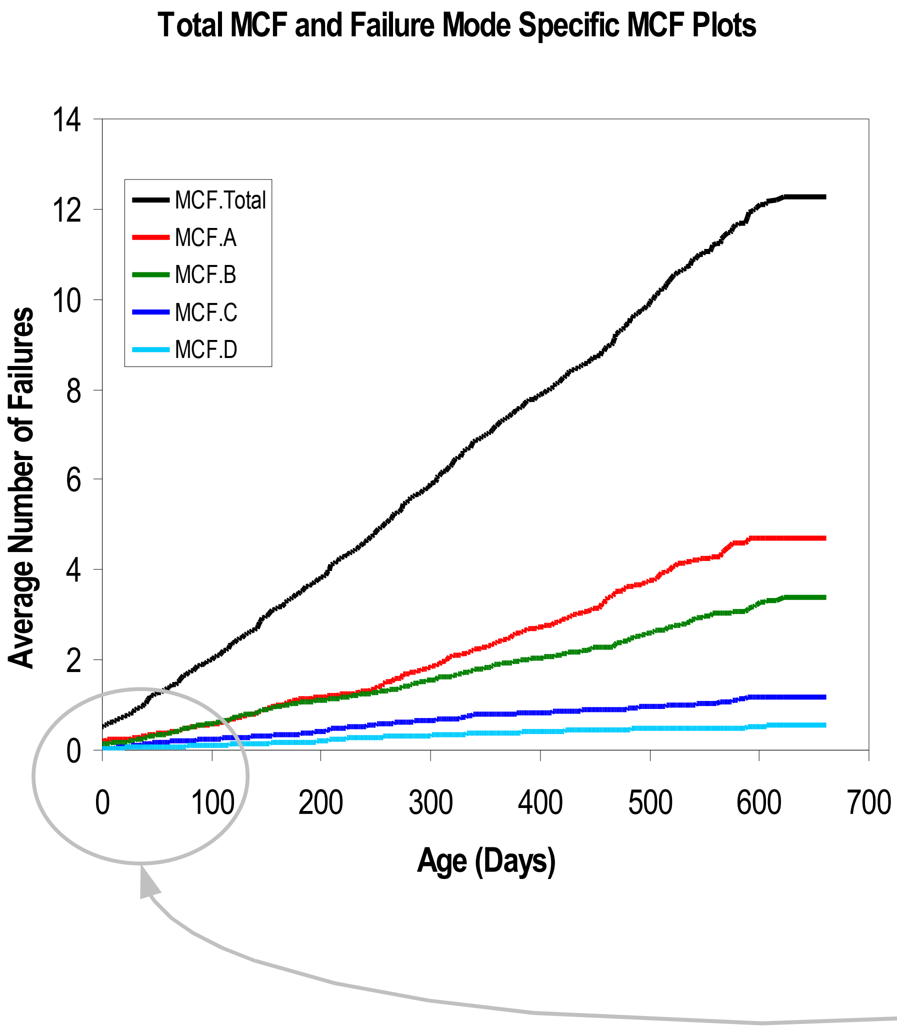
- In *TDR* analysis, we typically view the data in two forms.
- ***Age dependency*** looks at failure causes related to the age of the system.
- ***Date dependency*** shows causes related to events associated with a specific date. Possible causes include software updates, general maintenance actions, physical relocation of systems, sudden environmental changes, and so on.

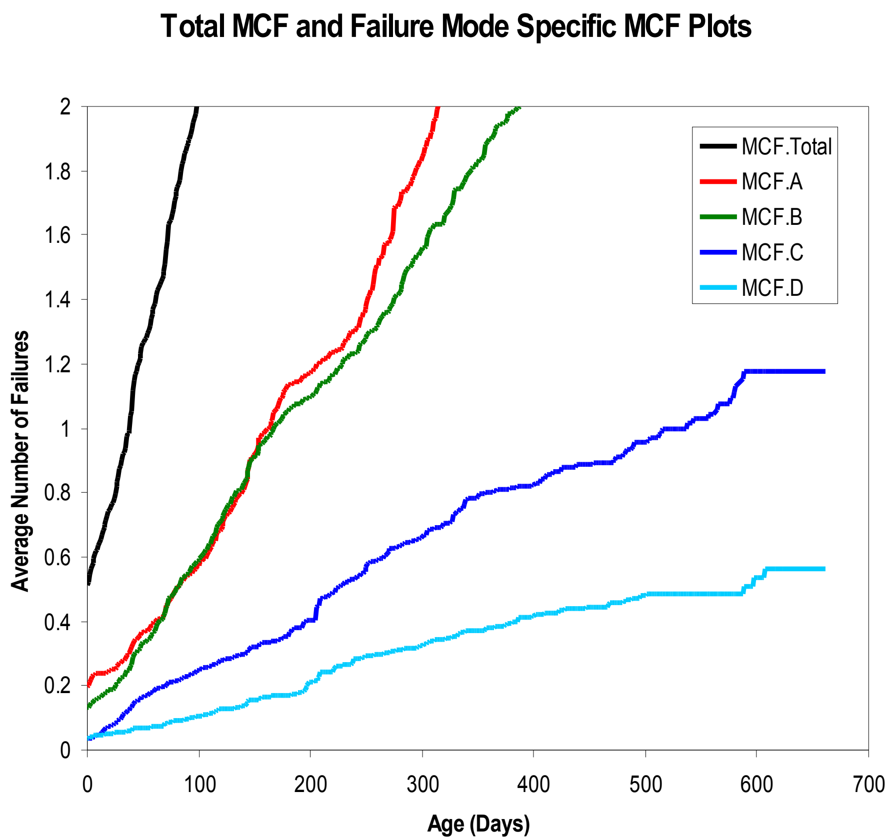
# Active Systems Plots



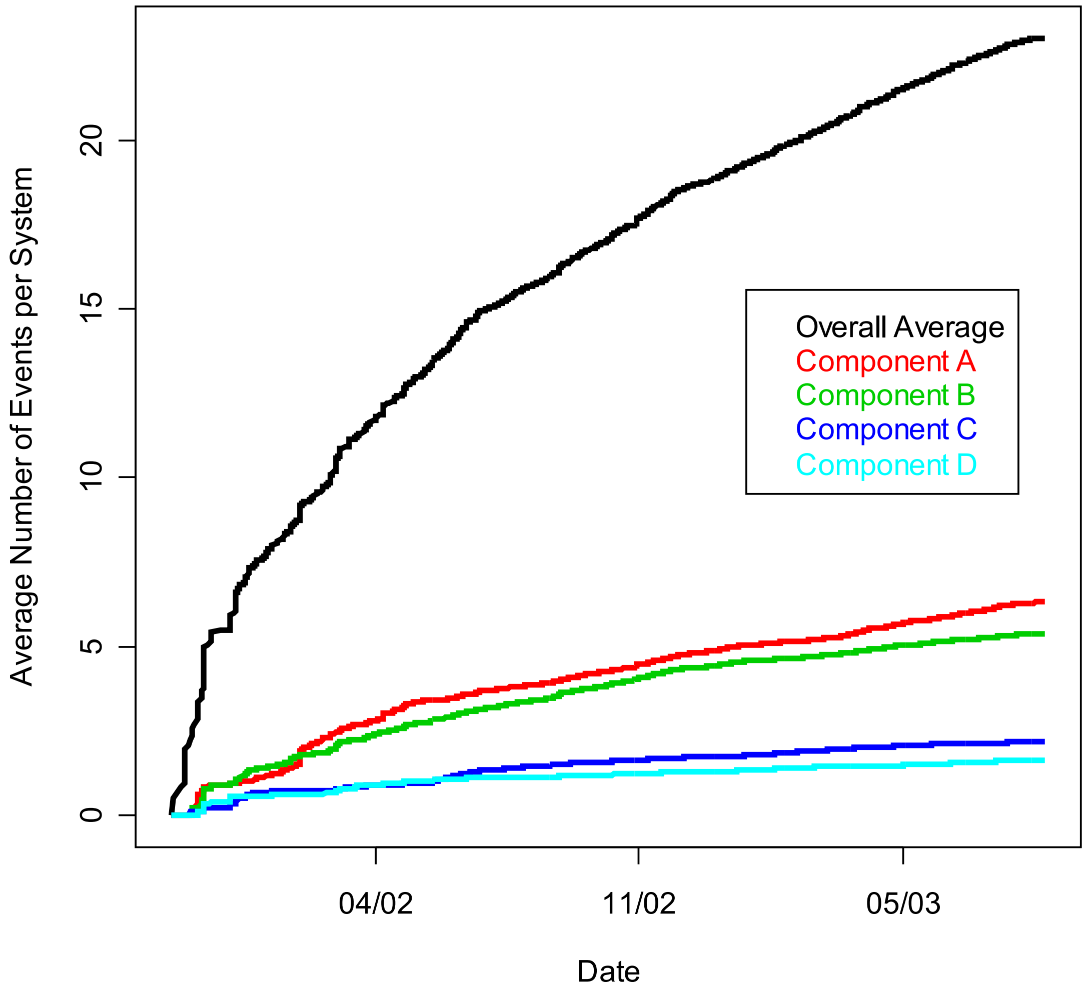



# MCF Plots by Age



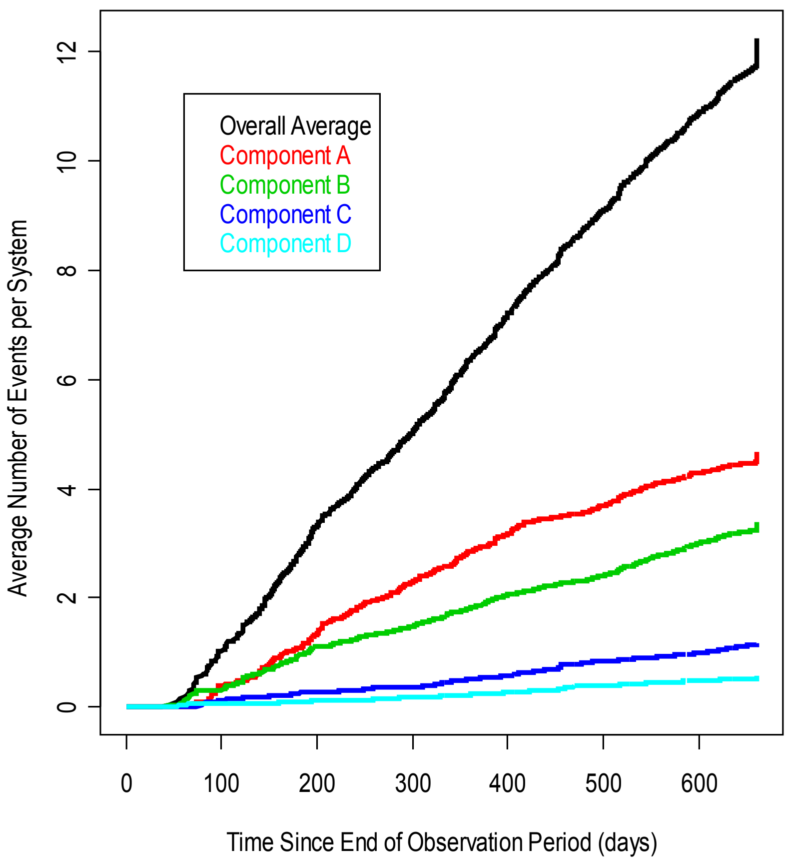
# MCF Plots by Date

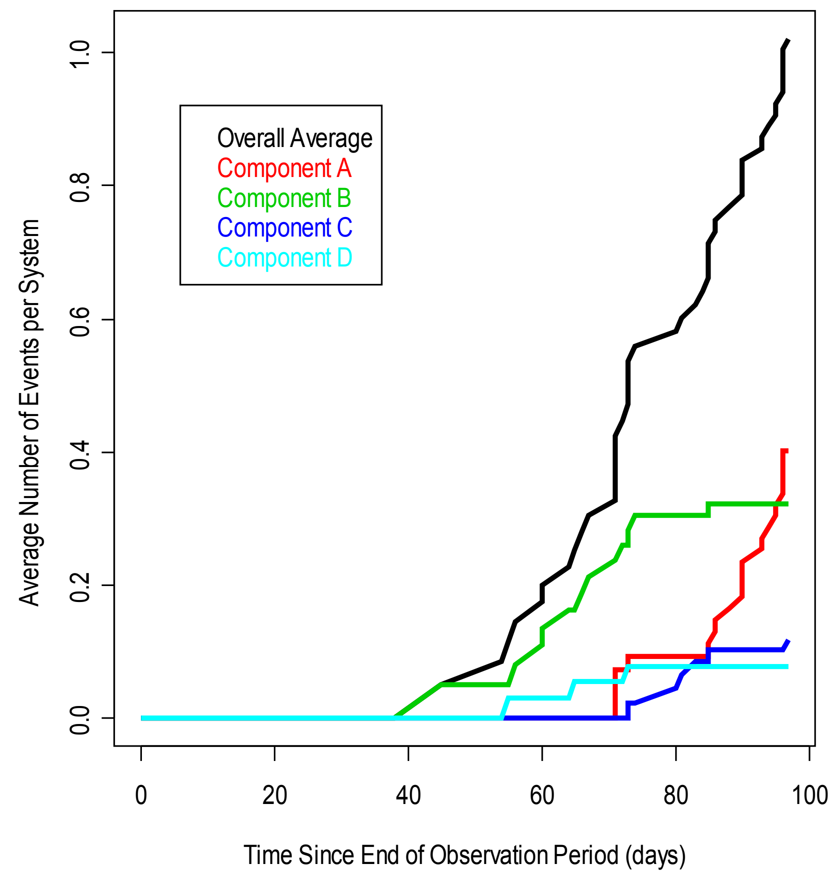


# *MCF* Plot with Reversed Time

- The *MCF* plot can also be reportrayed with the time axis reversed and the *MCF* curves shifted.
  - Shows most recent events to the left side of the graph.
  - Graph emphasizes to a greater extent the differences in failure modes in the later ages associated with the most recent events.
  - Reference time zero point used is oldest age.

# Reverse Time Plots



# *NHPP* Power Law Model for *MCF* Vs. Age

- Power law model for MCF:

$$M(t) = \alpha t^{\beta}$$

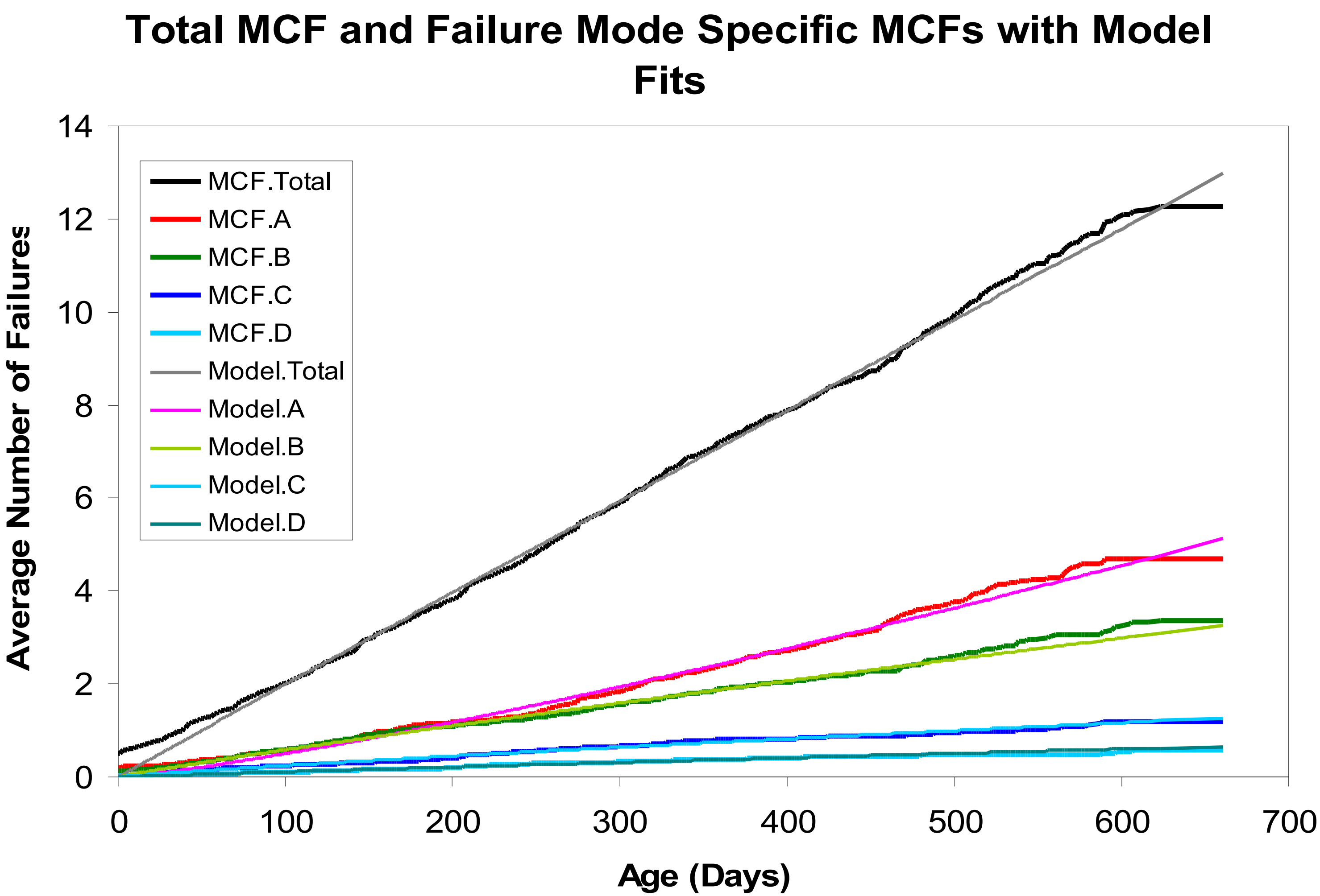
- Intensity function (theoretical recurrence rate)

$$\lambda(t) = dM(t)/dt = \alpha\beta t^{\beta-1}$$

# Parameter Estimation for Power Law Model

- Although *MLE* methods exist for estimating the parameters of the Power Law model (see Crow, 1974), a simple and direct approach is to estimate the parameters by minimizing the sum of squares of the residuals between the *MCF* and the model.
- The non-linear least squares ***Solver*** routine in the spreadsheet program EXCEL easily facilitates the estimation.

# Power Law Model Fits



# Parameters of Power Law Model

| | $\alpha$ | $\beta$ |
|---|---|---|
| • Overall MCF | .0207 | .991 |
| • Component A | .00175 | 1.23 |
| • Component B | .00887 | .910 |
| • Component C | .00455 | .868 |
| • Component D | .00148 | .940 |

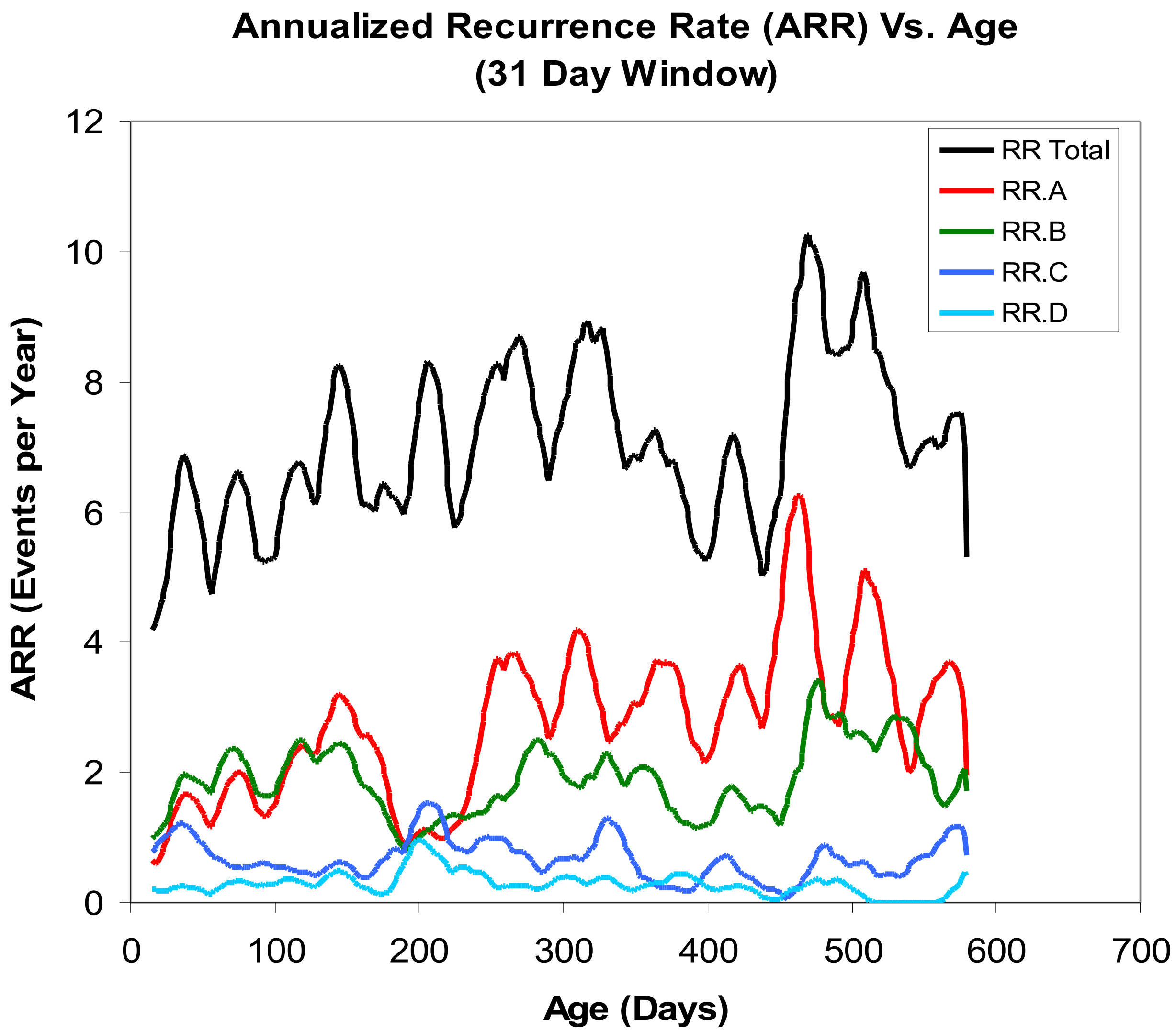
# Projections Using the Power Law Model

- With the estimated parameters, it is possible to use models to do overall or failure mode specific projections for future ages or dates.
- Such projections are a key component of warranty analysis, for example, to estimate spare parts inventories and staffing levels for field support.

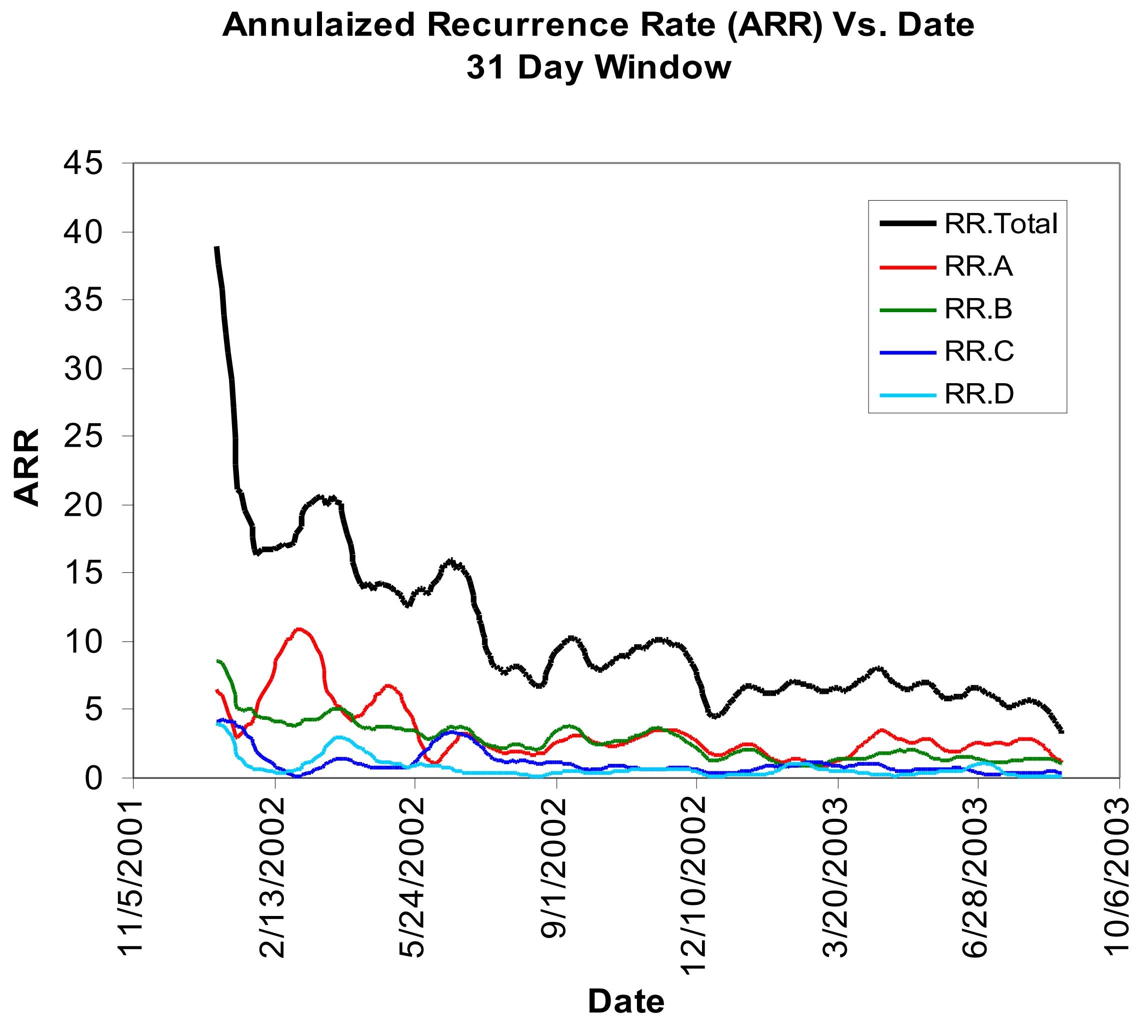
# Empirical Recurrence Rates (*RR*)

- It is possible to numerically differentiate the *MCF* curve and determine recurrence rates as a function of the system age or calendar date.
- A simple approach to the recurrence rate calculations is possible using the built-in spreadsheet SLOPE function.
- A slope is found for a window (odd) number of consecutive *MCF* points and plotted at the median age of the points. Then, the first point is dropped and another consecutive point added, with the process repeated. (See Trindade, 1975)

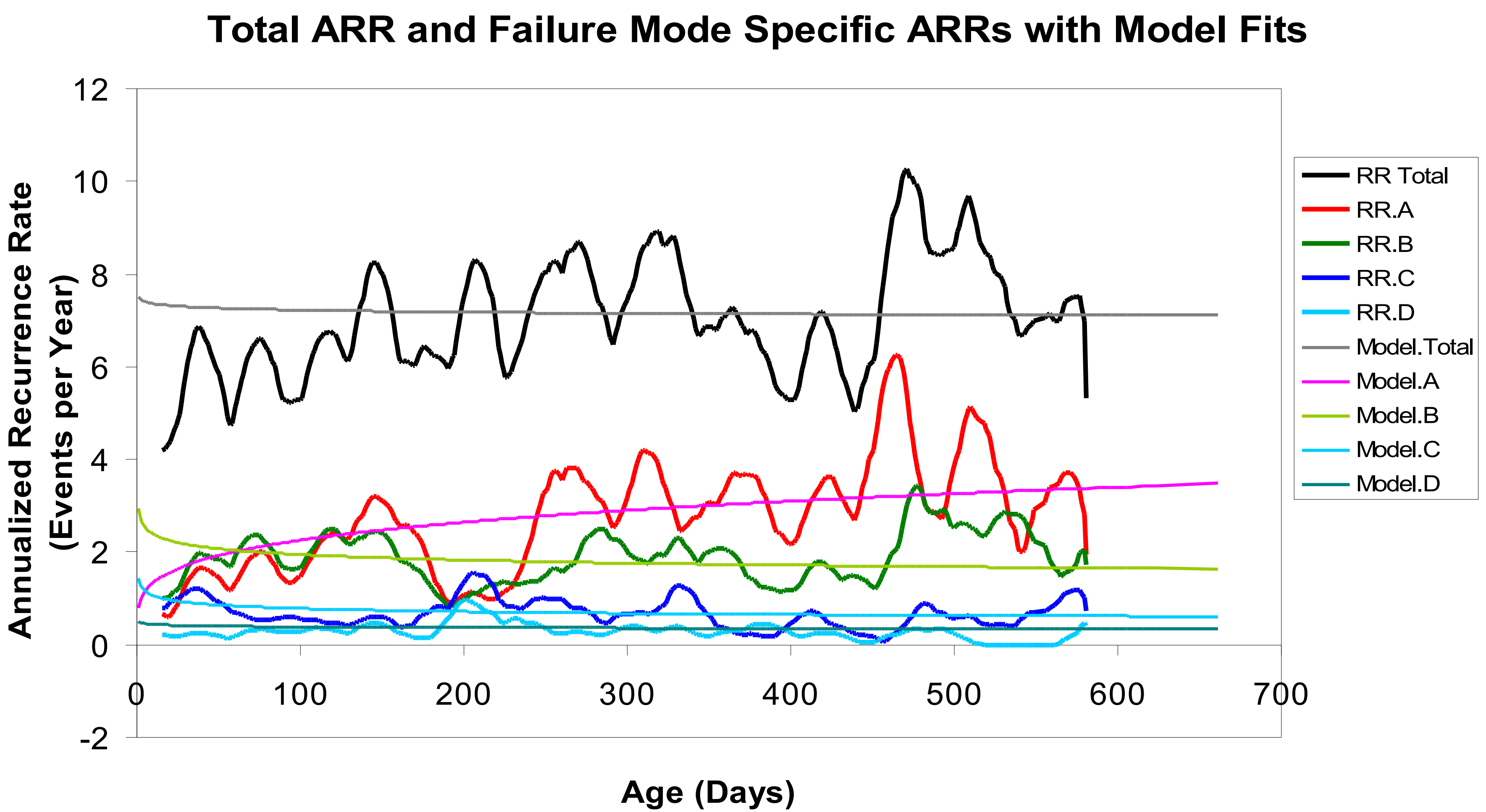
# *ARR* (Empirical) Vs. Age



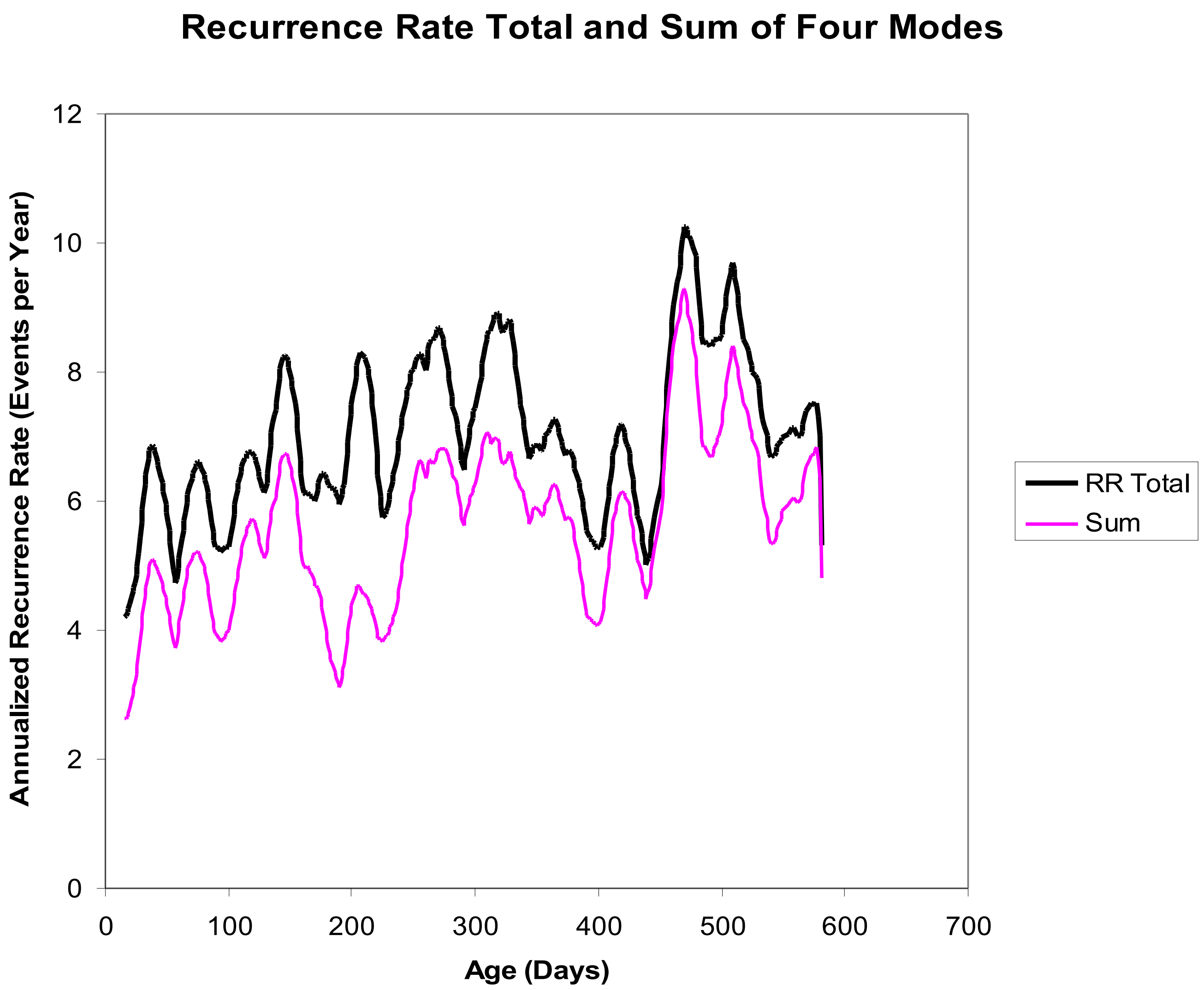
# Annualized Empirical Recurrence Rates (*ARR*) Vs. Date



# Annualized Recurrence Rates and Power Law Model Fits



# Total *RR* Vs. Failure Mode *RR* Sum

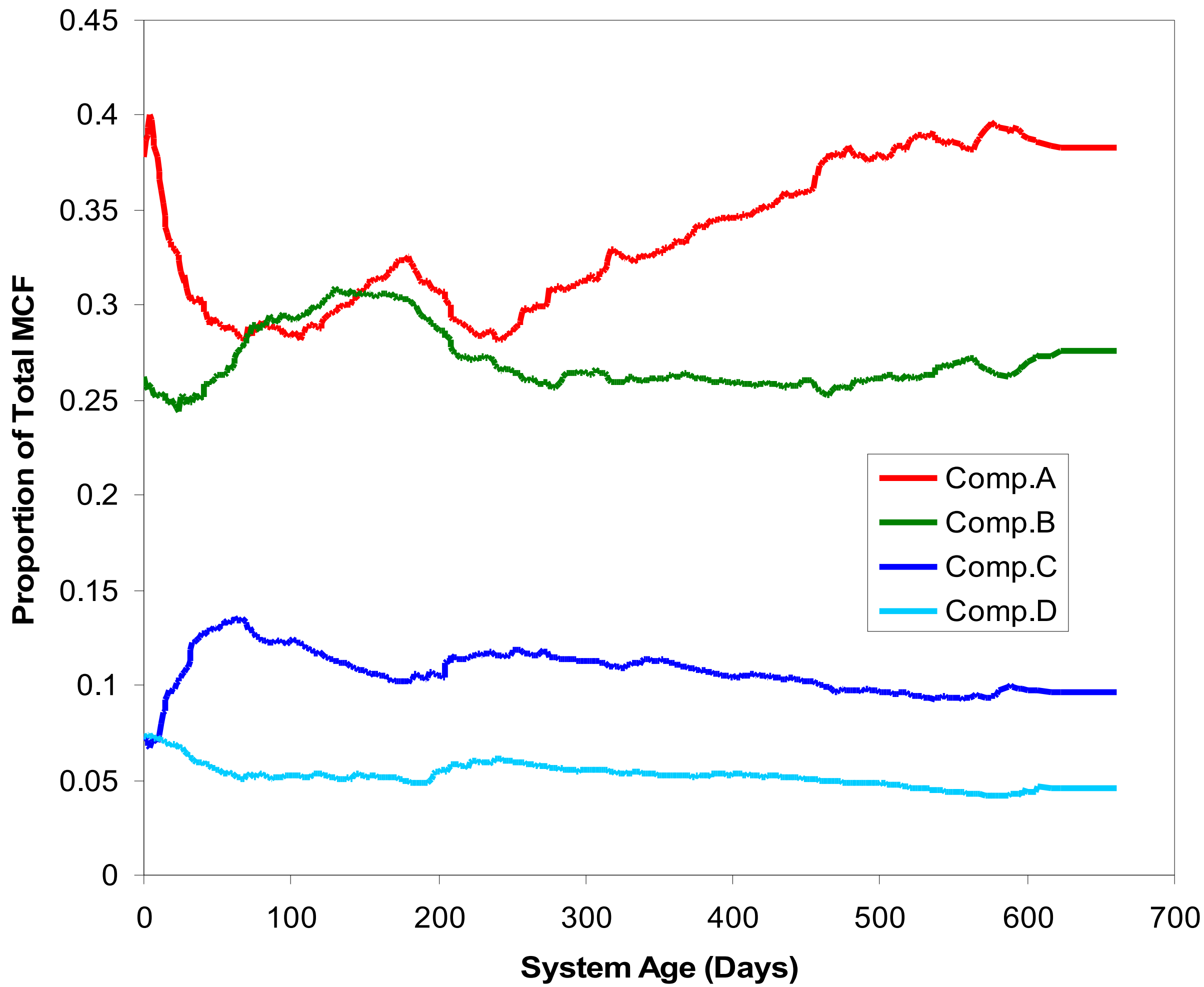


# Dynamic Pareto

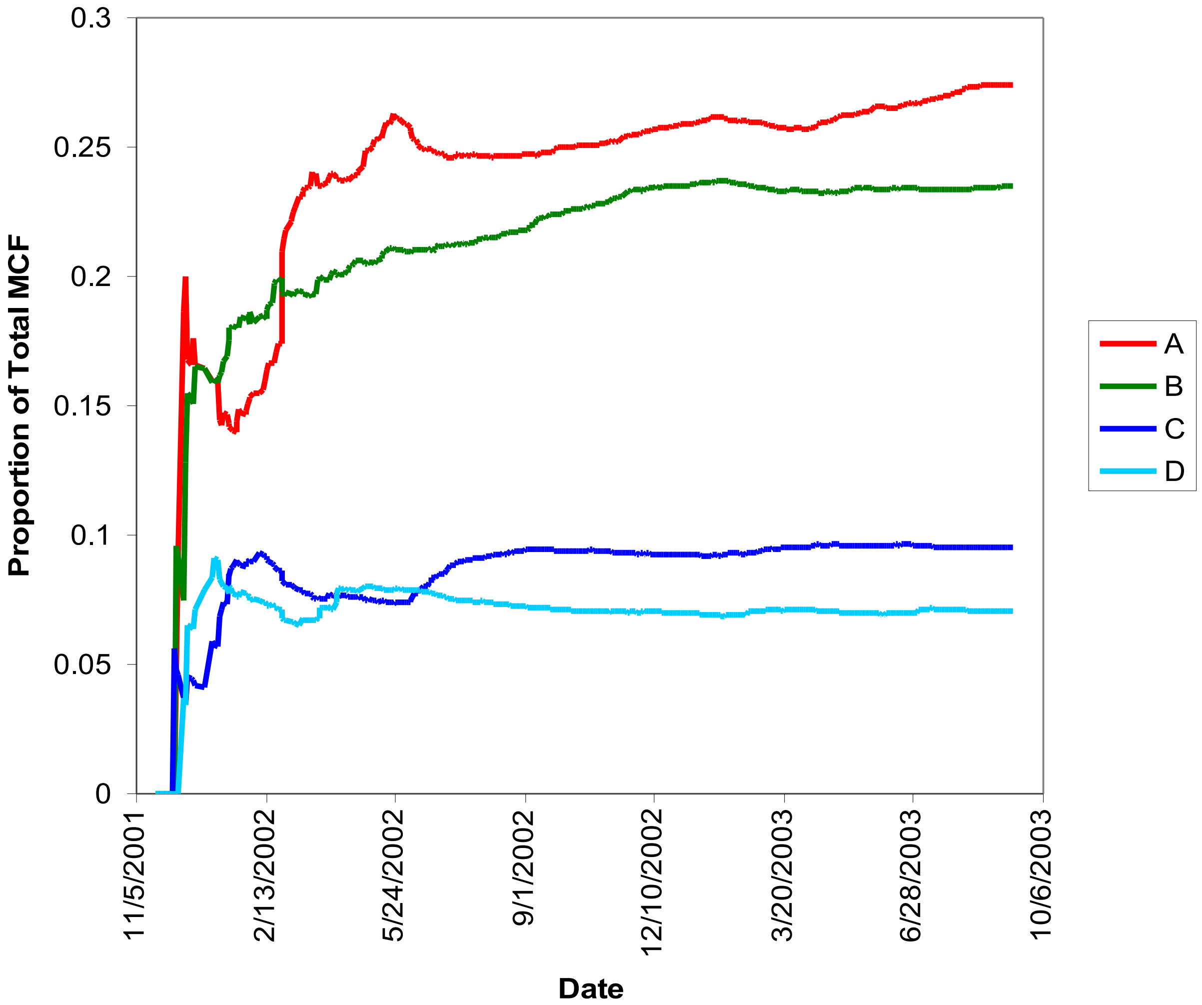
- The idea of the ***dynamic Pareto*** can be made easier to interpret by rescaling the curves.
- For the components, take the ratio of the component *MCF* to the overall *MCF*, e.g, for component *A*

$$R_A(t_k) = \frac{MCF_A(t_k)}{MCF_{Total}(t_k)}$$

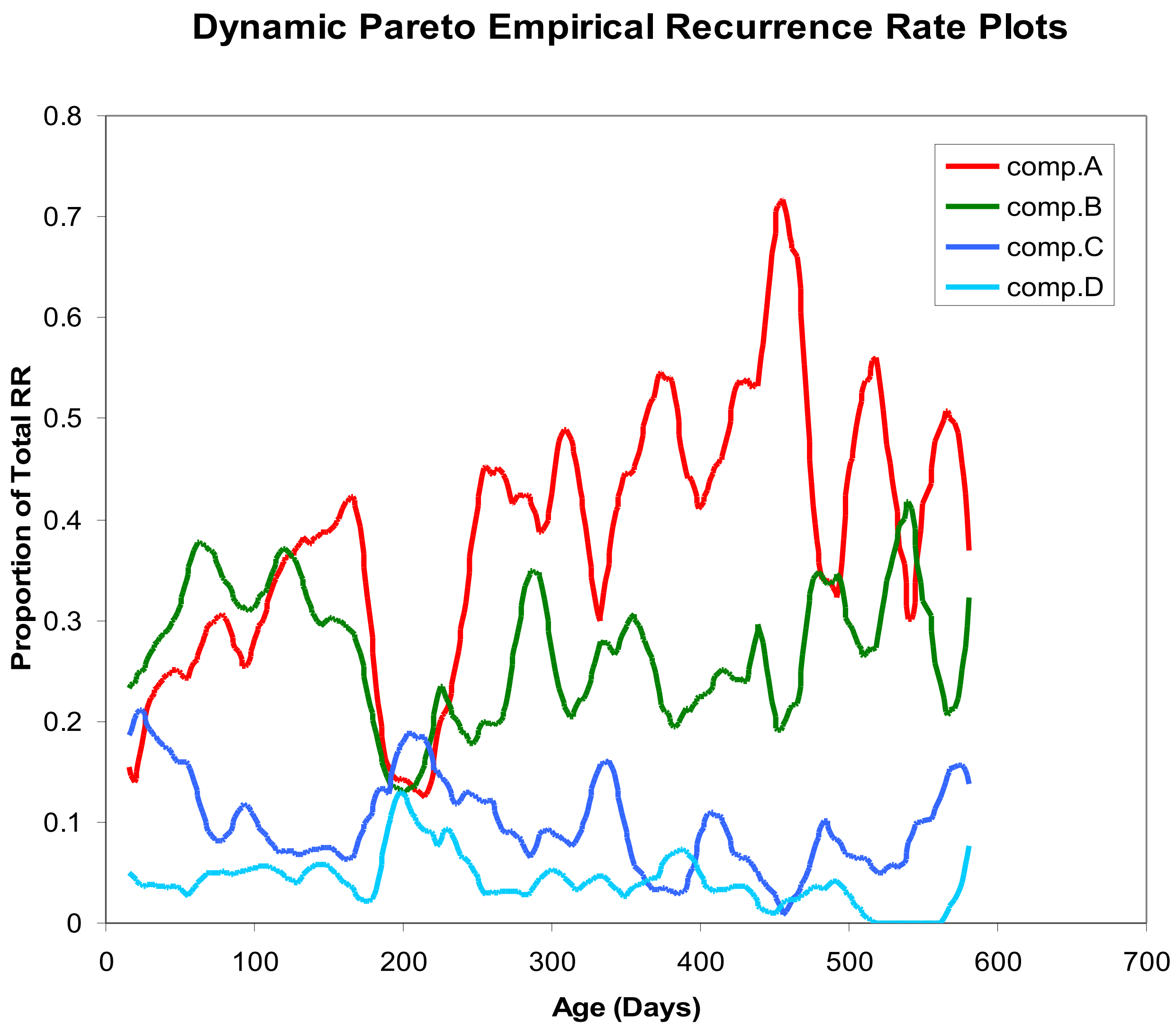
# Dynamic MCF Pareto by Age



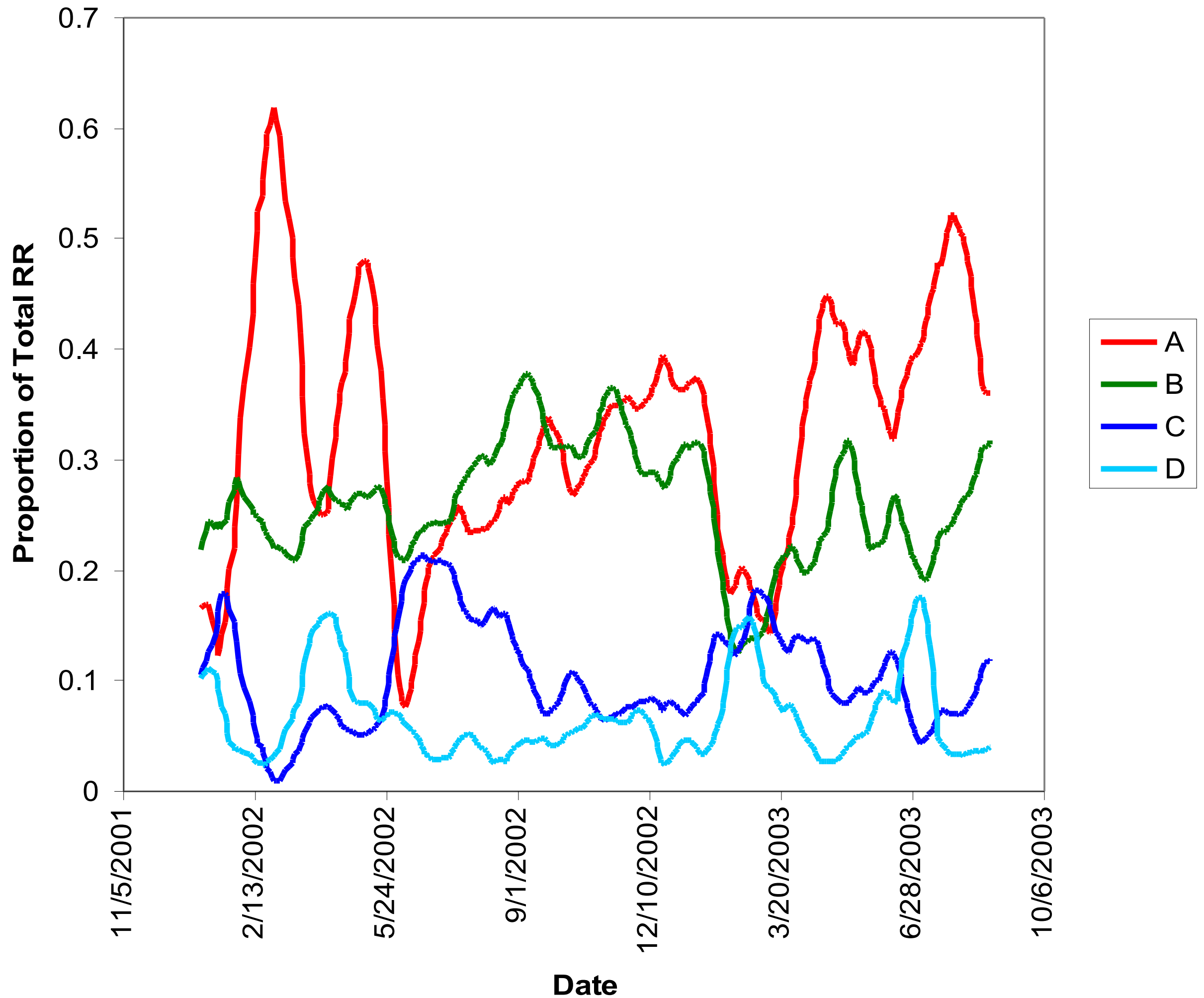
# Dynamic MCF Pareto by Date



# Dynamic RR Pareto by Age



# Dynamic RR Pareto by Date



# Average Cost per Unit Time

- The Cost per Unit Time computed from the *MCF*:

$$MCF(t_0) = 0$$

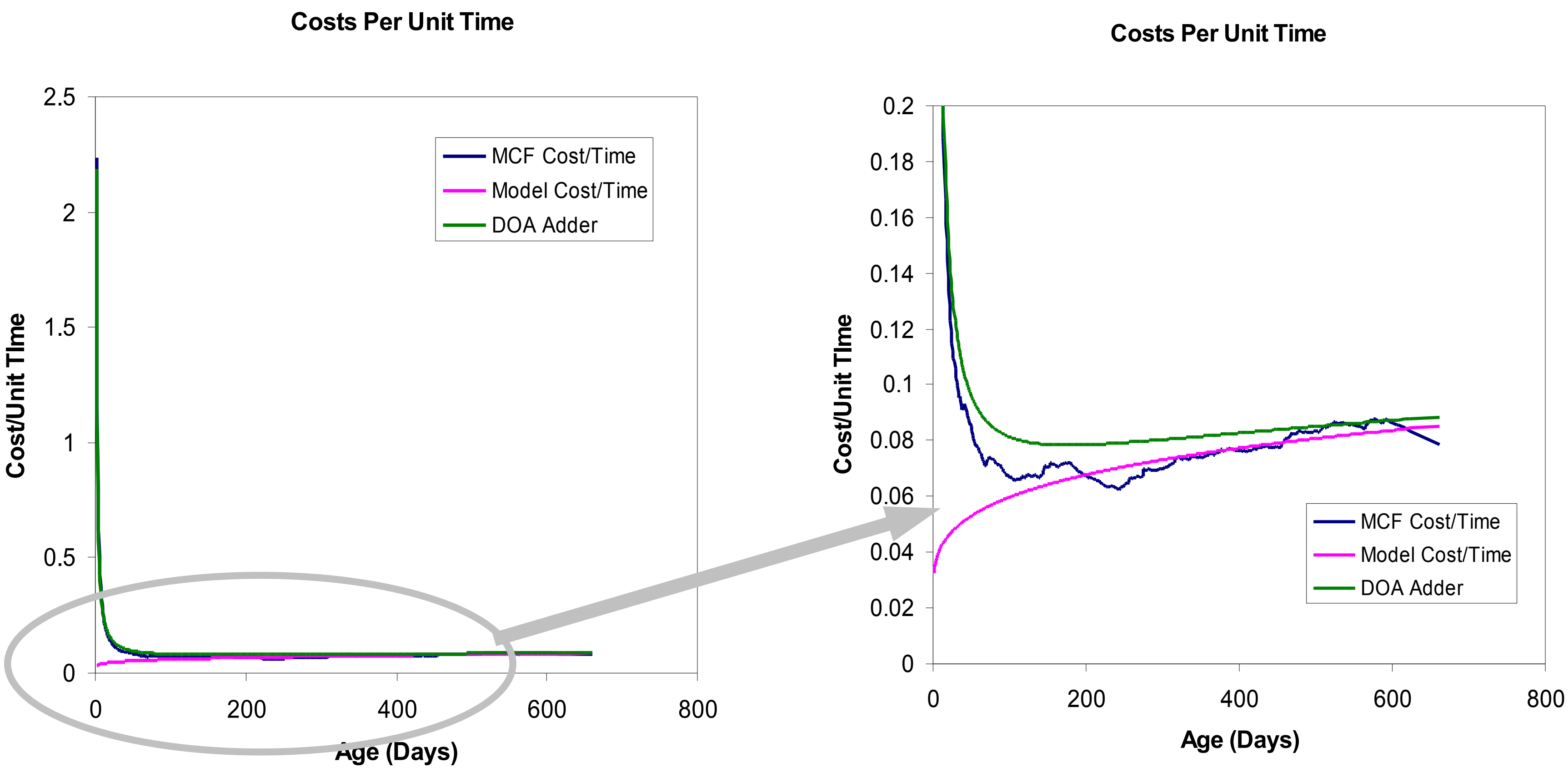
$$MCF(t_k) = MCF(t_{k-1}) + \frac{\sum_m c_m n(m, t_k)}{N(t_k)}$$

Cost per Unit Time = $MCF(T)/(T - t_0)$, where

$n(m,t_k)$ = number of fails of failure mode $m$ at time $t_k$,

$c_m$ = (marginal) cost of repairing failure mode $m$.

# Cost Per Unit Time Plots




Assigned Cost Weights: A(10), B(1), C(1), D(1)

# *MCF* Analysis Benefits

- MCFs are close to the data and so one can do data-sensitive calculations.
- Benefits shows up with:
  - (a) the failure-mode-specific MCFs
  - (b) calendar MCFs
  - (c) reverse-time/recent failure plotting
  - (d) the dynamic Pareto plots
- MCFs work with large populations & MCFs work with small populations
  - The empirical approach is revealing with cost-per-unit-time plots, too, and goodness-of-fit plots of power law models.

# Dynamic Pareto and Visualization

- The dynamic Pareto concept is very useful in the analysis of warranty claims.
- Both MCFs and RRs can reveal interesting aspects of the data using the dynamic Pareto approach.
- These visualization methods are easy to grasp and provide excellent insight into specific failure causes that vary with age or date.
- In most cases, the analysis can be easily accomplished using simple spreadsheet programs.
- Power law model complements *MCF* & *RR* analysis.

# References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